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COMPUTATIONAL
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A dynamically parameterized inversion-free iteration for a system of nonlinear matrix equation

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Abstract. Computation of the stabilizing solution pair of a system of nonlinear matrix equations is of great interest in calculating the Green's function of nanoparticles. By noting that each solution of the pair might have various sizes, an inversion-free iteration with dynamical parameters is proposed in this paper. Under proper assumptions the convergence of the algorithm is established, as well as the bound of the iteration sequence. Preliminary numerical experiments indicate that the dynamically parameterized inversion-free iteration is very efficient to compute the stabilizing solution pair.

Key words: dynamical parameters, inversion-free, fixed-point iteration, system of nonlinear matrix equation.

1. INTRODUCTION

Consider a system of nonlinear matrix equations (SNME)

$$\begin{cases} X + A^\top Y^{-\alpha} A = I_n, \\ Y + B^\top X^{-\beta} B = I_m \end{cases} \tag{1}$$

with $X \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{m\times m}$, $A, B \in \mathbb{R}^{m\times n}$ and $0 < \alpha, \beta \le 1$. Such a system has a special application to the computation of the Green's function of nanoparticles [1–3], which could be described by a semi-infinite periodic Hamiltonian system

$$\mathscr{H}_B = \begin{pmatrix} \mathscr{H}_P & \mathscr{H}_{PL} & & \\ \mathscr{H}_{PL}^\top & \mathscr{H}_P & \mathscr{H}_{PL} & \\ & \mathscr{H}_{PL}^\top & \mathscr{H}_P & \ddots \\ & & \ddots & \ddots \end{pmatrix}$$

with

$$\mathscr{H}_P = \begin{pmatrix} H_{b_1} & H_{b_1 b_2} & & \\ H_{b_1 b_2}^\top & H_{b_2} & \ddots & \\ & \ddots & \ddots & H_{b_{l-1} b_l} \\ & & H_{b_{l-1} b_l}^\top & H_{b_l} \end{pmatrix}$$

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and

$$\mathscr{H}_{PL} = \begin{pmatrix} & 0 & \ddots \\ H_{b_l b_1} & & \end{pmatrix}.$$

Here, for $t = 1, ..., l$, $H_{b_t} \in \mathbb{R}^{n_t \times n_t}$ stands for the t-th particle which contains interactions between all orbitals within identically adjacent layers and $H_{b_t b_{t+1}} \in \mathbb{R}^{n_t \times n_{t+1}}$ represents the interactions between the nearest neighbour particles H_{b_t} and $H_{b_{t+1}}$. In particular, $H_{b_l b_{l+1}} := H_{b_l b_1}$ means the interactions between the neighbour particles H_{b_l} and H_{b_1}. Taking, for example, the period $l = 2$ (denoted by $n_1 = n$ and $n_2 = m$), the computation of the Green's function [4,5] then requires determining of the most northwest block matrix of the inverse of

$$\begin{pmatrix} I_n - H_{b_1} & -H_{b_1 b_2} & & & \\ -H_{b_1 b_2}^\top & I_n - H_{b_2} & -H_{b_2 b_1} & & \\ & -H_{b_2 b_1}^\top & I_n - H_{b_1} & -H_{b_1 b_2} & \\ & & \ddots & \ddots & \ddots \end{pmatrix}$$

and

$$\begin{pmatrix} I_m - H_{b_2} & -H_{b_2 b_1} & & & \\ -H_{b_2 b_1}^\top & I_m - H_{b_1} & -H_{b_1 b_2} & & \\ & -H_{b_1 b_2}^\top & I_m - H_{b_2} & -H_{b_2 b_1} & \\ & & \ddots & \ddots & \ddots \end{pmatrix},$$

which, denoted by G_{b_1} and G_{b_2}, respectively, fulfill the equations

$$\begin{aligned} G_{b_1} &= (I_n - H_{b_1} - H_{b_1 b_2} G_{b_2} H_{b_1 b_2}^\top)^{-1}, \\ G_{b_2} &= (I_m - H_{b_2} - H_{b_2 b_1} G_{b_1} H_{b_2 b_1}^\top)^{-1}. \end{aligned} \tag{2}$$

If only the interactive effects between different adjacent particles are considered (i.e. $H_{b_1} = 0$ and $H_{b_2} = 0$) by setting

$$X = G_{b_1}^{-1},\ A = H_{b_1 b_2}^\top,\ Y = G_{b_2}^{-1},\ B = H_{b_2 b_1}^\top,$$

the equations (2) constitute a system of nonlinear matrix equations (1) with $\alpha = \beta = 1$.

It is obvious in this case that the system (1) could be uniformed as

$$Z + C^\top Z^{-1} C = I_{n+m} \tag{3}$$

by incorporating matrices

$$C = \begin{pmatrix} 0 & B \\ A & 0 \end{pmatrix} \in \mathbb{R}^{(n+m)\times(n+m)},\ \ Z = \begin{pmatrix} X & 0 \\ 0 & Y \end{pmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}.$$

What physicists interested in real applications seek is the stabilizing solution (X, Y) satisfying $\rho(X^{-1}B) < 1$ and $\rho(Y^{-1}A) < 1$ with X and Y positive definite, the symbol $\rho(\cdot)$ being here the spectral radius. The existence of the stabilizing solution for (3) could be widely studied [4–9]. For the generalized system (1), readers are referred to [10–13] and their reference on the existence of a stabilizing solution.

Different iterative methods have been extensively applied to compute the stabilizing solution of the uniformed Eq. (3). Engwerda et. al. employed a simple fixed-point iteration [7], which was subsequently modified to an inversion-free version by Zhan [14]. Guo and Lancaster [15] accelerated the inversion-free version and presented a detailed convergence analysis. Other inversion-free iteration formats could be referred to [11,16] and a version of a convergent factor was added by [17]. Huang et. al [10] applied the inversion-free scheme into SNME (1) and devised two kinds of fixed-point iterations. For iteration methods with higher convergence of computing (3) including cyclic reduction, doubling algorithm, and Newton

method, see [8,18–22] as well as references therein. In this paper, by noting that X and Y might be of different sizes and the varying parameters (as iteration progresses) are helpful to accelerate the convergence, a dynamically parameterized inversion-free iteration is proposed. Under proper conditions, the convergence of the proposed algorithm is constructed and numerical experiments illustrate its effectiveness.

Throughout this paper, it is written $A \geq B$ ($A > B$) for symmetric matrices A and B if $A-B$ is a symmetric positive semidefinite (definite) matrix. $\rho(A)$ denotes here the spectral radius of the matrix A. Several lemmas are required in this paper.

Lemma 1 (Parodi [23]). *If $A > B > 0$ (or $A \geq B > 0$), then $A^\alpha > B^\alpha > 0$ (or $A^\alpha \geq B^\alpha > 0$) for all $0 < \alpha \leq 1$, and $0 < A^\alpha < B^\alpha$ (or $0 < A^\alpha \leq B^\alpha$) for all $-1 \leq \alpha < 0$.*

Lemma 2 (Zhan [14]). *If A and B are symmetric matrices of the same dimension with $B > 0$, then $ABA + B^{-1} \geq 2A$.*

Lemma 3 (Bhatia [24]). *If $0 < a \leq 1$, and A and B are positive definite matrices of the same dimension with A, $B \geq bI > 0$ and b is some positive constant, then*

$$\|A^a - B^a\| \leq ab^{a-1}\|A-B\|, \quad \|A^{-a} - B^{-a}\| \leq ab^{-(a+1)}\|A-B\|$$

with $\|\cdot\|$ being some matrix norm.

The rest of this paper is organized as follows. Several inversion-free iterations are reviewed and the dynamically parameterized inversion-free iteration is proposed in Section 2. Under proper assumptions, the convergence of the presented method is constructed in Section 3. Section 4 is devoted to the numerical experiments to show the effectiveness of the dynamically parameterized inversion-free iteration and Section 5 presents the conclusion.

2. DYNAMICALLY PARAMETERIZED INVERSION-FREE ITERATION

The simplest fixed-point iterative algorithm for solving NME (3) is

$$Z_{k+1} = I - C^\top Z_k^{-1} C, \quad Z_0 = I,$$

and its convergence was given in [7]. Zhan suggested an inversion-free version [14] with $Z_0 = W_0 = I$ via replacing Z_k^{-1} with W_k

$$\begin{cases} Z_{k+1} = I - C^\top W_k C, \\ W_{k+1} = 2W_k - W_k Z_k W_k. \end{cases}$$

To accelerate the convergence, Guo and Lancaster [15] set the initial as $0 < W_0 \leq I$, $X_0 = I$ and employed the new-derived W_k as

$$\begin{cases} W_{k+1} = 2W_k - W_k Z_k W_k, \\ Z_{k+1} = I - C^\top W_{k+1} C, \end{cases}$$

which could be further rewritten in a form of W_k as

$$W_{k+1} = 2W_k - W_k(I - C^\top W_k C)W_k. \tag{4}$$

Another inversion-free iteration proposed by El-Sayed and Al-Dbiban [16] is

$$\begin{cases} W_{k+1} = W_k - Z_k W_k + I, \\ Z_{k+1} = I - C^\top W_{k+1} C \end{cases} \quad \text{or} \quad \begin{cases} W_{k+1} = W_k - W_k Z_k + I, \\ Z_{k+1} = I - C^\top W_{k+1} C \end{cases}$$

with $W_0 = Z_0 = I$. Similarly, this format could be implemented only with W_k as

$$W_{k+1} = I + \frac{1}{2}(W_k C^\top W_k C + C^\top W_k A W_k). \tag{5}$$

Huang et. al [10] applied the above formats (4) and (5) in SNME (1) and devised the iterations

$$\begin{cases} \Phi_{k+1} = 2\Phi_k - \Phi_k(I - A^\top \Psi_k^\alpha A)\Phi_k, \\ \Psi_{k+1} = 2\Psi_k - \Psi_k(I - B^\top \Phi_{k+1}^\beta B)\Psi_k \end{cases} \tag{6}$$

and

$$\begin{cases} \Phi_{k+1} = I + \frac{1}{2}(\Phi_k A^\top \Psi_k^\alpha A + A^\top \Psi_k^\alpha A \Phi_k), \\ \Psi_{k+1} = I + \frac{1}{2}(\Psi_k A^\top \Phi_{k+1}^\beta A + A^\top \Phi_{k+1}^\beta A \Psi_k), \end{cases} \tag{7}$$

respectively, with $\Phi_0 = \Psi_0 = I$. They also constructed the following monotone convergence.

Theorem 1. *Let (X,Y) be the stabilizing solution pair of SNME* (1)*. Then the sequence $\{\Phi_k, \Psi_k\}$ generated by the iteration* (6) *with $\Phi_0 = \Psi_0 = I$ is well defined and satisfies $\Phi_0 < \Phi_1 \le \Phi_2 \le ... \le \Phi_k \le ... \le \Phi$, $\Psi_0 < \Psi_1 \le \Psi_2 \le ... \le \Psi_k \le ... \le \Psi$ and*

$$\lim_{k\to\infty} \Phi_k = \Phi, \quad \lim_{k\to\infty} \Psi_k = \Psi$$

with $\Phi = X^{-1}$ and $\Psi = Y^{-1}$.

Essentially, the two above inversion-free iterations are different fixed-point formats for the system

$$\begin{cases} \Phi + \Phi A^\top \Psi^\alpha A \Phi = \Phi^2, \\ \Psi + \Psi B^\top \Phi^\beta B \Psi = \Psi^2. \end{cases} \tag{8}$$

Note that the stabilizing solution pair (Φ, Ψ) might have various sizes and a dynamical factor might be helpful to enhance the convergence rate. We generalize the iteration (6) to the scheme

$$\begin{cases} \Phi_{k+1} = (1+\gamma_k)\Phi_k - \gamma_k \Phi_k(I - A^\top \Psi_k^\alpha A)\Phi_k, \\ \Psi_{k+1} = (1+\delta_k)\Psi_k - \delta_k \Psi_k(I - B^\top \Phi_{k+1}^\beta B)\Psi_k \end{cases} \tag{9}$$

with $\Phi_0 = \Psi_0 = I$, $\gamma_k > 0$ and $\delta_k > 0$, referred to as the Dynamically Parameterized Inversion-free Iteration (DPII).

Obviously, DPII (9) reduces to the iteration (6) when $\gamma_k = \delta_k = 1$ and the iteration sequence (Φ_k, Ψ_k) will be monotonically convergent as dictated by Theorem 1. Unfortunately, if both γ_k and δ_k are merely positively selected, the monotonicity, even the convergence of the iteration sequence (Φ_k, Ψ_k) might not occur. It will be seen from the analysis of the convergence in the next section that the appropriate γ_k and δ_k are supposed to have a limit of 1. It means, if setting $\gamma_k = 1 + \varepsilon_k$ and $\delta_k = 1 + \bar{\varepsilon}_k$, then $lim_{k\to\infty}\varepsilon_k = 0, lim_{k\to\infty}\varepsilon_k = 0$. This facilitates a required assumption about ε_i and $\bar{\varepsilon}_i$ to obtain the bound and the convergence of the DPII sequence.

Lemma 4. *Let (Φ_k, Ψ_k) be the sequence generated by DPII for $k \ge 0$. Let (Φ, Ψ) be the solution of the system* (8)*. Suppose for any $k \ge 0$ that $\sum_{i=0}^k O(\varepsilon_i)$ and $\sum_{i=0}^k O(\bar{\varepsilon}_i)$ are sufficiently small, then $\Phi_1 \le \Phi + O(\varepsilon_0)$, $\Psi_1 \le \Psi + O(\varepsilon_0) + O(\bar{\varepsilon}_0)$ and for $k \ge 2$*

$$\Phi_k \le \Phi + \sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-2} O(\bar{\varepsilon}_i), \;\; \Psi_k \le \Psi + \sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-1} O(\bar{\varepsilon}_i),$$

where $O(\varepsilon_i)$ and $O(\bar{\varepsilon}_i)$ stand for some symmetric matrix of the order ε_i and $\bar{\varepsilon}_i$, respectively.

Proof. Obviously, the system (8) has a solution pair satisfying $\Phi > I$ and $\Psi > I$ by Theorem 1. Starting with $\Phi_0 = \Psi_0 = I$, it follows from the DPII iteration (9), Lemma 1 and Lemma 2 that

$$\begin{aligned}\Phi_1 &= I + \gamma_0 A^\top A \\ &= 2I - (I - A^\top A) + O(\varepsilon_0) \\ &\le (I - A^\top A)^{-1} + O(\varepsilon_0) \\ &< (I - A^\top \Psi^\alpha A)^{-1} + O(\varepsilon_0) \\ &= \Phi + O(\varepsilon_0)\end{aligned}$$

and

$$\begin{aligned}\Psi_1 &= I + \delta_0 B^\top \Phi_1^\beta B \\ &\le I + \delta_0 B^\top (\Phi + O(\varepsilon_0))^\beta B \\ &= I + \delta_0 B^\top \Phi^\beta B + O(\varepsilon_0) \\ &= 2I - (I - B^\top \Psi^\beta B) + O(\varepsilon_0) + O(\bar{\varepsilon}_0) \\ &= \Psi + O(\varepsilon_0) + O(\bar{\varepsilon}_0),\end{aligned}$$

where we use a fact $(1+\varepsilon)^a = 1 + a\varepsilon + o(\varepsilon)$ with sufficient small ε and $a > 0$.

Suppose the conclusion holds true for the integer j, i.e.

$$\Phi_j \le \Phi + \textstyle\sum_{i=0}^{j-1} O(\varepsilon_i) + \sum_{i=0}^{j-2} O(\bar{\varepsilon}_i), \;\; \Psi_j \le \Psi + \sum_{i=0}^{j-1} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i).$$

It follows from the DPII iteration and the assumption that

$$\begin{aligned}\Phi_{j+1} &\le (1+\gamma_j)\Phi_j - \gamma_j \Phi_j \Big(I - A^\top \Big(\Psi + \textstyle\sum_{i=0}^{j-1} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \Big)^\alpha A \Big) \Phi_j \\ &= (1+\gamma_j)\Phi_j - \gamma_j \Phi_j (I - A^\top \Psi^\alpha A)\Phi_j + \textstyle\sum_{i=0}^{j-1} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \\ &= (1-\gamma_j)\Phi_j + \gamma_j (2\Phi_j - \Phi_j \Phi^{-1} \Phi_j) + \textstyle\sum_{i=0}^{j-1} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \\ &\le \Phi + \varepsilon_j (\Phi - \Phi_j) + \textstyle\sum_{i=0}^{j-1} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \\ &= \Phi + \textstyle\sum_{i=0}^{j} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i)\end{aligned}$$

and

$$\begin{aligned}\Psi_{j+1} &\le (1+\delta_j)\Psi_j - \delta_j \Psi_j \Big(I - B^\top \Big(\Phi + \textstyle\sum_{i=0}^{j} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \Big)^\beta B \Big) \Psi_j \\ &= (1+\delta_j)\Psi_j - \delta_j \Psi_j (I - B^\top \Phi^\beta B)\Psi_j + \textstyle\sum_{i=0}^{j} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \\ &= (1-\delta_j)\Psi_j + \delta_j (2\Psi_j - \Psi_j \Psi^{-1} \Psi_j) + \textstyle\sum_{i=0}^{j} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \\ &\le \Psi + \bar{\varepsilon}_j (\Psi - \Psi_j) + \textstyle\sum_{i=0}^{j} O(\varepsilon_i) + \sum_{i=0}^{j-1} O(\bar{\varepsilon}_i) \\ &= \Psi + \textstyle\sum_{i=0}^{j} O(\varepsilon_i) + \sum_{i=0}^{j} O(\bar{\varepsilon}_i).\end{aligned}$$

The proof is completed by induction. □

The above theorem implies that the sequences $\{\Phi_k\}$ and $\{\Phi_k\}$ are bounded above if $\sum_{i=0}^{\infty} O(\varepsilon_i) < \infty$ and $\sum_{i=0}^{\infty} O(\bar{\varepsilon}_i) < \infty$. Moreover, we have the following corollary.

Corollary 1. *For $k \ge 0$, both $I - A^\top \Psi_k^\alpha A$ and $I - B^\top \Phi_k^\beta B$ are positive definite.*

Proof. We only show $I - A^\top \Psi_k^\alpha A$ is positive definite and the proof for $I - B^\top \Phi_k^\beta B$ is similar. In fact, it follows from Lemma 4 that

$$\begin{aligned}I - A^\top \Psi_k^\alpha A &> I - A^\top \Big(\Psi + \textstyle\sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-1} O(\bar{\varepsilon}_i) \Big)^\alpha A \\ &= I - A^\top \Psi^\alpha A + \textstyle\sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-1} O(\bar{\varepsilon}_i) \\ &= \Phi^{-1} + \textstyle\sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-1} O(\bar{\varepsilon}_i).\end{aligned}$$

Since $\sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-1} O(\bar{\varepsilon}_i)$ is sufficiently small by the assumption and Φ is positive definite, $I - A^\top \Psi_k^\alpha A$ is positive definite. □

3. CONVERGENCE ANALYSIS

To show the convergence of DPII, a lemma is first required.

Lemma 5. *Let (Φ_k, Ψ_k) be the sequence generated by DPII for $k \geq 0$. Then, under the assumptions of Lemma 4 we have $\Phi_2 > \Phi_1 + O(\varepsilon_0) + O(\bar{\varepsilon}_0)$ and $\Psi_2 > \Psi_1 + O(\varepsilon_0) + O(\bar{\varepsilon}_0)$ and for $k \geq 2$*

$$\begin{aligned}\Phi_{k+1} &> \Phi_k + \textstyle\sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-2} O(\bar{\varepsilon}_i),\\ \Psi_{k+1} &> \Psi_k + \textstyle\sum_{i=0}^{k-1} O(\varepsilon_i) + \sum_{i=0}^{k-1} O(\bar{\varepsilon}_i),\end{aligned}$$

where $O(\varepsilon_0)$ and $O(\bar{\varepsilon}_0)$ stand for some symmetric matrix of the order ε_0 and $\bar{\varepsilon}_0$, respectively.

Proof. Starting with $\Phi_0 = \Psi_0 = I$, it follows from DPII (9) that

$$\begin{aligned}\Phi_1 &= I + (1+\varepsilon_0)A^\top A > \Phi_0 + O(\varepsilon_0),\\ \Psi_1 &= I + (1+\bar{\varepsilon}_0)B^\top \Phi_1^\beta B > \Psi_0 + O(\varepsilon_0) + O(\bar{\varepsilon}_0).\end{aligned}$$

For sufficiently small $O(\varepsilon_0) + O(\bar{\varepsilon}_0)$, it has

$$\begin{aligned}\Phi_1 &= \gamma_0\Big(2\Phi_0 - \Phi_0(I - A^\top \Psi_0^\alpha A)\Phi_0\Big) - \varepsilon_0\Phi_0\\ &\leq \gamma_0(I - A^\top\Psi_0^\alpha A)^{-1} + O(\varepsilon_0)\\ &< \gamma_0\Big(I - A^\top\Psi_1^\alpha A + O(\varepsilon_0) + O(\bar{\varepsilon}_0)\Big)^{-1} + O(\varepsilon_0)\\ &= \gamma_0(I - A^\top\Psi_1^\alpha A)^{-1} + O(\varepsilon_0) + O(\bar{\varepsilon}_0).\end{aligned}$$

Since Ψ_1 has an upper bound by Lemma 4, then $\Phi_1 < (I - A^\top\Psi_1^\alpha A)^{-1} + O(\varepsilon_0) + O(\bar{\varepsilon}_0)$ and thus

$$\begin{aligned}\Phi_1^{-1} &> \Big((I - A^\top\Psi_1^\alpha A)^{-1} + O(\varepsilon_0) + O(\bar{\varepsilon}_0)\Big)^{-1}\\ &= I - A^\top\Psi_1^\alpha A + O(\varepsilon_0) + O(\bar{\varepsilon}_0).\end{aligned}$$

Therefore,

$$\begin{aligned}\Phi_2 - \Phi_1 &= \gamma_1\Phi_1\Big(\Phi_1^{-1} - (I - A^\top\Psi_1^\alpha A)\Big)\Phi_1\\ &> \gamma_1\Phi_1\Big(O(\varepsilon_0) + O(\bar{\varepsilon}_0)\Big)\Phi_1\\ &= O(\varepsilon_0) + O(\bar{\varepsilon}_0).\end{aligned}$$

Analogously, it follows from DPII (9) again that

$$\begin{aligned}\Psi_1 &= \delta_0\Big(2\Psi_0 - \Psi_0(I - B^\top\Phi_1^\beta B)\Psi_0\Big) - \bar{\varepsilon}_0\Psi_0,\\ &\leq \delta_0(I - B^\top\Phi_1^\beta B)^{-1} + O(\bar{\varepsilon}_0)\\ &< \delta_0\Big(I - B^\top\Phi_2^\beta B + O(\varepsilon_0) + O(\bar{\varepsilon}_0)\Big)^{-1} + O(\bar{\varepsilon}_0)\\ &= \delta_0(I - B^\top\Phi_2^\beta B)^{-1} + O(\varepsilon_0) + O(\bar{\varepsilon}_0).\end{aligned}$$

As Φ_2 is bounded above by Lemma 4, then $\Psi_1 < (I - B^\top\Phi_2^\beta B)^{-1} + O(\varepsilon_0) + O(\bar{\varepsilon}_0)$ and thus

$$\begin{aligned}\Psi_1^{-1} &> \Big((I - B^\top\Phi_2^\beta B)^{-1} + O(\varepsilon_0) + O(\bar{\varepsilon}_0)\Big)^{-1}\\ &= I - B^\top\Phi_2^\beta B + O(\varepsilon_0) + O(\bar{\varepsilon}_0).\end{aligned}$$

So,

$$\begin{aligned}\Psi_2-\Psi_1&=\delta_1\Psi_1\Big(\Psi_1^{-1}-(I-B^\top\Phi_2B)\Big)\Psi_1\\&>\delta_1\Psi_1\Big(O(\varepsilon_0)+O(\bar{\varepsilon}_0)\Big)\Psi_1\\&=O(\varepsilon_0)+O(\bar{\varepsilon}_0).\end{aligned}$$

Following the above process and noting that $I-A^\top\Psi_2^\alpha A$ is nonsingular from Corollary 1, one can similarly show

$$\begin{aligned}&\Phi_2>\Phi_1+\textstyle\sum_{j=0}^{1}O(\varepsilon_j)+O(\bar{\varepsilon}_0),\\&\Psi_2>\Psi_1+\textstyle\sum_{j=0}^{1}O(\varepsilon_j)+\sum_{j=0}^{1}O(\bar{\varepsilon}_j).\end{aligned}$$

Now suppose the conclusion holds true for $k=i$, i.e.

$$\begin{aligned}&\Phi_{i+1}>\Phi_i+\textstyle\sum_{j=0}^{i-1}O(\varepsilon_j)+\sum_{j=0}^{i-2}O(\bar{\varepsilon}_j),\\&\Psi_{i+1}>\Psi_i+\textstyle\sum_{j=0}^{i-1}O(\varepsilon_j)+\sum_{j=0}^{i-1}O(\bar{\varepsilon}_j).\end{aligned}$$

It follows from Corollary 1 that $I-A^\top\Psi_{i+1}^\alpha A$ is nonsingular and thus

$$\begin{aligned}\Phi_{i+1}&=\gamma_i\Big(2\Phi_i-\Phi_i(I-A^\top\Psi_i^\alpha A)\Phi_i\Big)-\varepsilon_i\Phi_i\\&<\gamma_i\Big(I-A^\top\Big(\Psi_{i+1}+\textstyle\sum_{j=0}^{i-1}O(\varepsilon_j)+\sum_{j=0}^{i-1}O(\bar{\varepsilon}_j)\Big)^\alpha A\Big)^{-1}+O(\varepsilon_i)\\&<\gamma_i(I-A^\top\Psi_{i+1}^\alpha A)^{-1}+\textstyle\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i-1}O(\bar{\varepsilon}_j).\end{aligned}$$

Then $\Phi_{i+1}^{-1}>I-A^\top\Psi_{i+1}^\alpha A+\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i-1}O(\bar{\varepsilon}_j)$, which implies

$$\begin{aligned}\Phi_{i+2}-\Phi_{i+1}&=\gamma_{i+1}\Phi_{i+1}\Big(\Phi_{i+1}^{-1}-(I-A^\top\Psi_{i+1}^\alpha A)\Big)\Phi_{i+1}\\&>\gamma_{i+1}\Phi_{i+1}\Big(\textstyle\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i-1}O(\bar{\varepsilon}_j)\Big)\Phi_{i+1}.\end{aligned}$$

Note that Φ_{i+1} is bounded above, then $\Phi_{i+2}>\Phi_{i+1}+\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i-1}O(\bar{\varepsilon}_j)$. Repeating the above process for Ψ_{i+1} and using the induction assumption, it is not difficult to show

$$\Psi_{i+1}^{-1}>I-B^\top\Phi_{i+2}^\beta B+\textstyle\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i}O(\bar{\varepsilon}_j).$$

Then one has

$$\begin{aligned}\Psi_{i+2}-\Psi_{i+1}&=\delta_{i+1}\Psi_{i+1}\Big(\Psi_{i+1}^{-1}-(I-B^\top\Phi_{i+2}^\beta B)\Big)\Psi_{i+1}\\&>\delta_{i+1}\Psi_{i+1}\Big(\textstyle\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i}O(\bar{\varepsilon}_j)\Big)\Psi_{i+1}\\&=\textstyle\sum_{j=0}^{i}O(\varepsilon_j)+\sum_{j=0}^{i}O(\bar{\varepsilon}_j),\end{aligned}$$

completing the proof by induction. □

Theorem 2. *Let (Φ_k,Ψ_k) be the sequence generated by DPII (9) for $k\geq 0$. Let (Φ,Ψ) be the symmetric positive solution pair of the system* (8)*. If $\|\Psi^\alpha A\|\|\Phi^\beta B\|$ are small enough and the selected γ_k and δ_k such that $\|I-\gamma_k\Phi^{-1}\Phi_k\|$ and $\|I-\delta_k\Psi^{-1}\Psi_k\|$ are sufficiently small, then the sequence (Φ_k,Ψ_k) of DPII is convergent to (Φ,Ψ).*

Proof. For each k, it follows from Lemma 3 and Lemma 5 that there exits $a_k > 0$ and $b_k > 0$ such that

$$\|\Psi^{-\alpha}\Psi_k^{\alpha}\Psi^{-\alpha} - \Psi^{-\alpha}\| \le \alpha a_k^{\alpha-1}\|\Psi^{-1}\Psi_k\Psi^{-1} - \Psi^{-1}\|,$$
$$\|\Phi^{-\beta}\Phi_k^{\beta}\Phi^{-\beta} - \Phi^{-\beta}\| \le \beta b_k^{\beta-1}\|\Phi^{-1}\Phi_k\Phi^{-1} - \Phi^{-1}\|.$$

From the DPII format, one has

$$\begin{aligned}
&\|\Phi^{-1}\Phi_{k+1}\Phi^{-1} - \Phi^{-1}\| \\
&= \|(1+\gamma_k)\Phi^{-1}\Phi_k\Phi^{-1} - \Phi^{-1} - \gamma_k\Phi^{-1}\Phi_k(\Phi^{-1} + A^{\top}\Psi^{\alpha}A - A^{\top}\Psi_k^{\alpha}A)\Phi_k\Phi^{-1}\| \\
&= \|(I - \gamma_k\Phi^{-1}\Phi_k)(\Phi^{-1}\Phi_k\Phi^{-1} - \Phi^{-1}) + \gamma_k\Phi^{-1}\Phi_k A^{\top}(\Psi_k^{\alpha} - \Psi^{\alpha})A\Phi_k\Phi^{-1}\| \\
&\le \|I - \gamma_k\Phi^{-1}\Phi_k\|\ \|\Phi^{-1}\Phi_k\Phi^{-1} - \Phi^{-1}\| + \alpha\gamma_k a_k^{\alpha-1}\|\Phi^{-1}\Phi_k\|^2\|\|\Psi^{\alpha}A\|^2 \\
&\quad \cdot\|\Psi^{-1}\Psi_k\Psi^{-1} - \Psi^{-1}\|.
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\|\Psi^{-1}\Psi_{k+1}\Psi^{-1} - \Psi^{-1}\| \\
&\le \|I - \delta_k\Psi^{-1}\Psi_k\|\ \|\Psi^{-1}\Psi_k\Psi^{-1} - \Psi^{-1}\| + \beta\delta_k b_k^{\beta-1}\|\Psi^{-1}\Psi_k\|^2\|\Phi^{\beta}B\|^2 \\
&\quad \cdot\|\Phi^{-1}\Phi_{k+1}\Phi^{-1} - \Phi^{-1}\| \\
&= \left(\alpha\beta\gamma_k\delta_k a_k^{\alpha-1}b_k^{\beta-1}\|\Phi^{-1}\Phi_k\|^2\|\Psi^{-1}\Psi_k\|^2\|\Phi^{\alpha}A\|^2\|\Psi^{\beta}B\|^2 + \|I - \delta_k\Psi^{-1}\Psi_k\|\right) \\
&\quad \cdot\|\Psi^{-1}\Psi_k\Psi^{-1} - \Psi^{-1}\| \\
&\quad + \beta\delta_k b_k^{\beta-1}\|I - \gamma_k\Phi^{-1}\Phi_k\|\|\Psi^{-1}\Psi_k\|^2\|\Phi^{\beta}B\|^2\ \|\Phi^{-1}\Phi_k\Phi^{-1} - \Phi^{-1}\|.
\end{aligned}$$

Combining the above two inequalities yields

$$\begin{pmatrix} \|\Phi^{-1}\Phi_{k+1}\Phi^{-1} - \Phi^{-1}\| \\ \|\Psi^{-1}\Psi_{k+1}\Psi^{-1} - \Psi^{-1}\| \end{pmatrix} \le M_k \begin{pmatrix} \|\Phi^{-1}\Phi_k\Phi^{-1} - \Phi^{-1}\| \\ \|\Psi^{-1}\Psi_k\Psi^{-1} - \Psi^{-1}\| \end{pmatrix}$$

with $M_k = (M_k^{ij})$, $i, j = 1, 2$ and thus

$$\begin{aligned}
M_k^{11} &= \|I - \gamma_k\Phi^{-1}\Phi_k\|, \\
M_k^{12} &= \alpha\gamma_k a_k^{\alpha-1}\|\Phi^{-1}\Phi_k\|^2\|\Psi^{\alpha}A\|^2, \\
M_k^{21} &= \beta\delta_k b_k^{\beta-1}\|I - \gamma_k\Phi^{-1}\Phi_k\|\|\Psi^{-1}\Psi_k\|^2\|\Phi^{\beta}B\|^2, \\
M_k^{22} &= \alpha\beta\gamma_k\delta_k a_k^{\alpha-1}b_k^{\beta-1}\|\Phi^{-1}\Phi_k\|^2\|\Psi^{-1}\Psi_k\|^2\|\Phi^{\alpha}A\|^2\|\Psi^{\beta}B\|^2 \\
&\quad + \|I - \delta_k\Psi^{-1}\Psi_k\|.
\end{aligned}$$

Since Φ_k and Ψ_k are bounded above, if $\|\Phi^{\alpha}A\|\|\Psi^{\beta}B\|$ is small enough and the selected γ_k and δ_k are such that $\|I - \gamma_k\Phi^{-1}\Phi_k\|$ and $\|I - \delta_k\Psi^{-1}\Psi_k\|$ are sufficiently small, one has $\rho(M_k) < 1$, showing that the sequence (Φ_k, Ψ_k) of DPII is convergent to (Φ, Ψ). □

Remark. (i). The condition on small $\|\Phi^{\alpha}A\|\|\Psi^{\beta}B\|$ for the convergence of DPII is similar to that (i.e., small $\|Z^{-1}C\|$) for the convergence of the fixed-point iteration in the uniformed Eq. (3) [4,5,14,18]. The smaller than 1, the faster the convergence of iteration. The choice of γ_k and δ_k is another factor affecting the convergence rate. It is seen that γ_k and δ_k are supposed to have the limit 1 when iteration converges, i.e., the smaller are $\|I - \gamma_k\Phi^{-1}\Phi_k\|$ and $\|I - \delta_k\Psi^{-1}\Psi_k\|$, the faster is the convergence of DPII.

(ii). The best optimal parameters γ_k and δ_k might not be obtained as Φ and Ψ are unavailable before iterations. There are several alternatives to an approximation: (1) $\gamma_k = \rho(\Phi_k^{-1}\Phi_{k-1})$ and $\delta_k = \rho(\Psi_k^{-1}\Psi_{k-1})$; (2) $\gamma_k = \|\Phi_{k-1}\|/\|\Phi_k\|$ and $\delta_k = \|\Psi_{k-1}\|/\|\Psi_k\|$; (3) $\gamma_k = \|\Phi_k^{-1}\Phi_{k-1}\|$ and $\delta_k = \|\Psi_k^{-1}\Psi_{k-1}\|$. As the computation of spectral radius in (1) is more expensive and the strategy (2) is observed to be prone to divergence when it iterates, the strategy (3) is preferred in our experiments where a conservative rule is selected, i.e., $\gamma_k = \min\{\max\{1, \|\Phi_k^{-1}\Phi_{k-1}\|\}, \zeta_1\}$ and $\delta_k = \min\{\max\{1, \|\Psi_k^{-1}\Psi_{k-1}\|\}, \zeta_2\}$ with ζ_1 and ζ_2 a little greater than 1.

4. NUMERICAL EXPERIMENTS

In this section, the effectiveness of the developed DPII is demonstrated for computing the stabilizing solution pair. The numerical experiments reported in [10] indicate that the iteration (6) (denoted here by IFI) has better performances than the iteration (7). So, the numerical comparison was merely conducted between DPII and the inversion-free iteration (IFI) on a PC with Intel i3-3240 3.4GHz processor and 8GB RAM, where both algorithms were coded by MATLAB 2014. Moreover, they were terminated when the relative residual satisfied $\|\mathrm{Resx}\| + \|\mathrm{Resy}\| < tol$ with

$$\mathrm{Resx} = \frac{\|\Phi_k^{-1} + A^\top \Psi_k^\alpha A - I_n\|}{\|\Phi_k^{-1}| \ + \|A\|^2 \|\Psi_k^\alpha\| \|I_n\|}, \quad \mathrm{Resy} = \frac{\|\Psi_k^{-1} + B^\top \Phi_k^\beta B - I_m\|}{\|\Psi_k^{-1}\| + \|B\|^2 \|\Phi_k^\beta\| \|I_m\|}$$

and the tolerance $tol = 10^{-12}$. We employed $\zeta_1 = \zeta_2 = 1.5$ in the strategy (3) and recorded the iteration number (It), iteration time (CPU), and the relative residual (Resx for the first equation and Resy for the second equation in the system) in tables when the algorithms terminated. In particular, the obtained relative residuals at each iteration were plotted as figures of residual history.

Example 1. This example is obtained from Example 2 in [10]. Consider SNME (1) with

$$A = \frac{1}{10}\begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 4 & 2 \\ 1 & 0 & 2 & 0 \end{pmatrix}, \ B = \frac{1}{10}\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{pmatrix}$$

and $\alpha = 0.95$, $\beta = 0.9$. When both algorithms are terminated, it is seen from Table 1 that DPII is able to attain the prescribed accuracy with fewer iterations than IFI did. The residual history in Fig. 1 also revealed that the residual level of DPII (red line) was nearly always below that of IFI (blue line) at each iteration, showing that DPII tended to arrive at a lower residual level.

Example 2. This example is obtained from Example 1 in [10] with proper modification to various dimensions of A and B. Consider SNME (1) with $A = Q_1^{-1/2} \tilde{A} Q_2^{-1/2} \in \mathbb{R}^{r_1 \times r_2}$, $B = Q_2^{-1/2} \tilde{B} Q_1^{-1/2} \in \mathbb{R}^{r_2 \times r_1}$, where $Q_1 = I + \tilde{A}\tilde{A}^\top \in \mathbb{R}^{r_1 \times r_1}$, $Q_2 = I + \tilde{B}^\top \tilde{B} \in \mathbb{R}^{r_2 \times r_2}$, and

$$(\tilde{A})_{ij} = \frac{4}{i+j-1} \in \mathbb{R}^{r_1 \times r_2}, \ \ (\tilde{B})_{ij} = \frac{1}{i+j-1} \in \mathbb{R}^{r_1 \times r_2}.$$

Positive constants $\alpha = 0.9$, $\beta = 0.9$ as well as the dimension $r_1 = 10$ and $r_2 = 100$ were employed to run IFI and DPII. Both algorithms stopped regularly and were able to derive the stabilizing solution pair. From the numerical results listed in Table 2, one can see that DPII attained the prescribed residual level within fewer iterations and less CPU time.

It is also interesting to observe from the residual history (i.e., Fig. 2) that the residual level of DPII decreased slower than that of IFI at the first 9 iterations. However, it dropped much lower than IFI did for the rest of the iterations.

Table 1. Numerical results for Example 1

Alg	It.	CPU	Resx	Resy
IFI	10	0.006	5.12e-13	8.95e-17
DPII	9	0.006	2.49e-13	1.01e-17

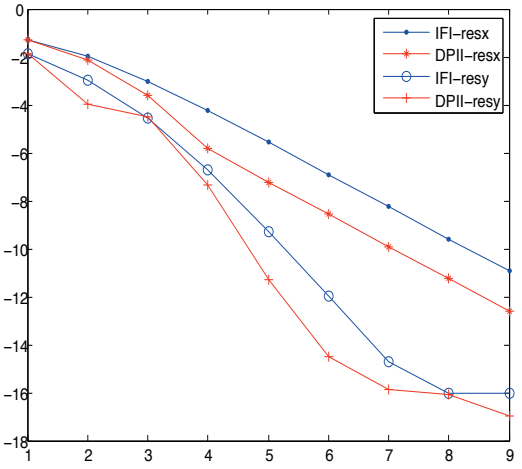

Fig. 1. Residual history for Example 1.

Table 2. Numerical results for Example 2

Alg	It.	CPU	Resx	Resy
IFI	23	0.445	5.54e-13	1.64e-16
DPII	20	0.412	7.08e-13	2.43e-16

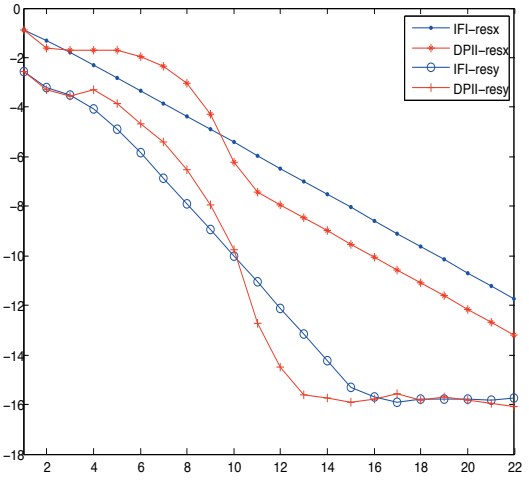

Fig. 2. Residual history for Example 2.

Table 3. Numerical results for Example 3

Alg	It.	CPU	Resx	Resy
IFI	13	0.259	3.67e-13	1.21e-16
DPII	12	0.234	2.95e-13	2.71e-16

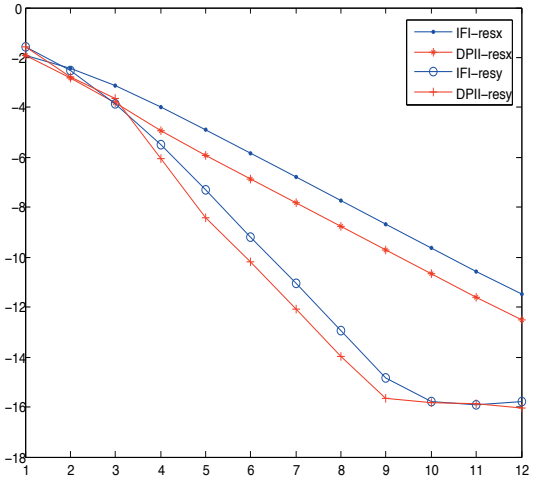

Fig. 3. Residual history for Example 3.

Example 3. Consider SNME with random-generated matrices $R_a \in \mathbb{R}^{r_1 \times r_2}$, $R_b \in \mathbb{R}^{r_1 \times r_2}$ and $r_1 = 10$, $r_2 = 100$. Set $\tilde{A} = 0.1 * R_a$, $\tilde{B} = 0.1 * R_b$ and construct Q_1, Q_2, A and B as in Example 2 but $\alpha = \beta = 1$, yielding

$$A = \begin{pmatrix} 0.01167 & \dots & -0.02098 \\ \vdots & \ddots & \vdots \\ 0.0358 & \dots & -0.02427 \end{pmatrix}, \quad B = \begin{pmatrix} -0.0060 & \dots & -0.0031 \\ \vdots & \ddots & \vdots \\ 0.00751 & \dots & 0.0149 \end{pmatrix}.$$

Both algorithms were run and the derived results recorded in Table 3, which indicates that DPII cost fewer iterations and less CPU time to arrive at the prescribed residual level. In addition, it can be seen from the residual history in Fig. 3 that the residual line of DPII is inclined to lie below that of IFI, except for the last several steps when computing "Resy".

5. CONCLUSIONS

A dynamically parameterized inversion-free iteration (DPII) is proposed in this paper for computing the stabilizing solution pair of a system of nonlinear matrix equations. The convergence of DPII is constructed under proper assumptions. Several strategies of selecting dynamical parameters are presented and the numerical experiments show that DPII is able to attain the prescribed residual level within fewer iterations and less CPU time than the existing inversion-free iterations. For future line of research, the choice of the most optimal dynamical parameters deserves more consideration.

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Dünaamiliselt parametriseeritud inversioonivaba iteratsioon mittelineaarse matriitsi võrrandi süsteemi jaoks

Ning Dong, Bo Yu ja Zhaoyun Meng

Mittelineaarse süsteemi stabiliseerimislahenduste paari arvutamise maatriksvõrrandid pakuvad reaalsetes rakendustes suurt huvi. Märkides, et paari kõik lahendused võivad olla erineva suurusega, on käesolevas artiklis välja pakutud dünaamiliste parameetritega inversioonivaba iteratsioon. Õigete eelduste korral tehakse kindlaks algoritmi lähenemine ja iteratsioonijärjestuse seotus. Esialgsed arvkatsed näitavad, et dünaamiliselt parametriseeritud inversioonivaba iteratsiooni abil on väga tõhus välja arvutada stabiliseeriva lahuse paar.