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CONTROL THEORY

Robust H∞ fault-tolerant control for stochastic Markov jump time-delay systems with actuator faults and application

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Abstract. This paper investigates the robust H∞ fault-tolerant controller design under actuator failure for a class of the stochastic Markov jump time-delay systems with parameter uncertainties. The existence condition of the state feedback robust H∞ fault-tolerant controller with actuator failure is presented. The robust H∞ fault-tolerant control algorithm is derived in the form of linear matrix inequality via the Lyapunov stability theory. The proposed control does not need to estimate the boundary value of an actuator fault, nor does it depend on fault detection and diagnostic devices. By solving the linear matrix inequality, a robust fault-tolerant controller, which makes the closed-loop system asymptotically stable and whose H∞ performance is restricted by a given bound, is designed such that its structure is comparably simpler and does not require a large number of calculations. The designed controller is applied to a UAV illustrative example. The numerical results and computer simulation demonstrate the effectiveness of the proposed fault-tolerant control.

Key words: actuator failure, Markov jump system, time-delay, robust fault-tolerant control, UAV.

1. INTRODUCTION

With the increasing complexity of the control systems, especially the systems with high safety requirements (such as aircraft, power systems, chemical facilities, nuclear energy facilities, etc.), the fault-tolerant control strategies need to be used in order to ensure that the system can still meet a certain stable performance when an abnormality occurs.

System integrity means that when one or more components in the system fail, the system can still work steadily by using the remaining components. In the early days, many scholars carried out research on this problem [1–3]. In 1971, Niederlinski proposed the concept of integral control [4], which is the idea of fault-tolerant control. If the closed-loop system is still stable and has ideal characteristics when the actuator, sensor or component fails, the closed-loop control system is called the fault-tolerant control system. Around 1980, Šiljak researched the problem of reliable stabilization of the system and published some results, which are the important early literature for the fault-tolerant control [5–7].

Faults in the engineering system mainly include the actuator fault, sensor fault, controller fault and controlled object fault [8–11]. The actuator is the most prone to failure because it performs control tasks frequently. The failure of the actuator in the system may cause the system to lose its original performance, or even cause the system to become unstable [12–15]. For example, in spacecraft control systems, the actuators are one of the key components for precise control. If the actuator fails, it will inevitably affect the performance of the spacecraft control system. In serious cases, it may even lead to the failure of the space mission. Therefore, when the actuators fail, how

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to use limited information to improve the stability of the system has attracted the attention of many scholars. In short, it is of great significance to study the robust fault-tolerant control when the system fails [16–18].

In recent years, many scholars have had some achievements in the research of robust fault-tolerant control [19–22]. Ma et al. [19] have investigated the networked non-fragile H_∞ control problem for Lipschitz nonlinear system with quantization and packet dropout in both feedback and forward channels. The problem of iterative learning of fault-tolerant control for multi-stage intermittent processes with uncertainties and actuator failures is studied in [20]. Tong et al. [21] have researched the adaptive fuzzy decentralized fault-tolerant control (FTC) problem for a class of nonlinear large-scale systems with strict feedback. The nonlinear system considered contains unmeasured states and actuator faults. By means of fuzzy logic systems, approximating unknown nonlinear functions, a fuzzy adaptive observer is designed to estimate the unmeasured state. Mahmoud and Khalid [22] have proposed for interconnected systems within the framework of integrated design a fault-tolerant control scheme to monitor and detect the faults in time, and to reconfigure the controller according to these faults.

In practical engineering systems, there is a wide range of systems with Markov chains. This system includes both time state evolution and event modal-driven hybrid dynamic systems. In particular, due to the existence of random phenomena such as component failures, changes in the external environment, and network delays, the systems may suddenly change in structure or parameters. At this time, the systems can often be abstracted as Markov jump systems for modelling and analysis. In recent years, many scholars have focused on Markov jump systems and have had some research achievements [23–29]. H_∞ state feedback control for singular Markov jump systems with incomplete transfer probability knowledge is studied in [23]. Zhang et al. [25] have designed a finite-time bounded observer with elasticity and robustness for a class of nonlinear systems with nonlinear measurement equations, which all have disappeared nonlinear model disturbances and additive perturbations. Moon and Başar [26] have considered the robust stochastic large population game for coupled Markov jump linear systems (MJLS). Based on the robust mean field game theory, a low complexity robust decentralized controller is designed. In the case of control for Markov jump time-delay systems, the category of control methodologies employing contemporary developments in switching BAS control as well as the switched systems theory are of considerable importance. Li et al. [28] have modelled the linear time-varying delay system with actuator failures as a switched linear time-varying delay system by utilizing the switched systems theory and a suitable control scheme. Li et al. [29] have considered the problem of reliable stabilization and H∞ control for a class of continuous-time switched Lipschitz nonlinear systems with actuator failures. The sufficient conditions for reliable exponential stabilization of the switched systems were derived by hybrid observer-based output feedback control.

The quad-rotor UAV is widely used in military reconnaissance, power inspection, aerial photography and in other fields due to its several advantages such as good stability, low flight speed, and low-altitude flight safety performance. In flight, a quad-rotor UAV may suffer from random disturbances such as the external environment changes, system parameters changes, and damage to the internal components of the system, which may result in faults. The Markov jump system model can effectively describe random mutations caused by failures and due to other reasons during system operation. Therefore, the quad-rotor UAV system model can be abstracted as a Markov jump system model description, and then the corresponding control method can be designed.

In this paper, for actuator failure the robust H∞ fault-tolerant control of stochastic Markov jump system with both state and input delays is studied. By establishing the fault model of the actuator, according to the Lyapunov stability theory, the sufficient condition for the existence of the robust H∞ fault-tolerant controller is given, which makes the closed-loop system asymptotically stable and meets certain H∞ interference suppression. The advantage of the robust fault-tolerant control designed in this paper lies in the fact that there is no need to estimate the boundary value of actuator failure, nor does it depend on fault detection and diagnostic devices. Finally, the designed controller is applied to a UAV illustrative example. The numerical results and computer simulation demonstrate the effectiveness of the proposed fault-tolerant control.

2. PROBLEM STATEMENTS

In the probability space (Ω, F, P), consider a stochastic Markov jump time-delay system with parameter uncertainties

$$\begin{cases} \dot{x}(t) = (A(r_t) + \Delta A(r_t))x(t) \\ +(A_d(r_t) + \Delta A_d(r_t))x(t-d_1) \\ +(B(r_t) + \Delta B(r_t))u(t) \\ +(B_d(r_t) + \Delta B_d(r_t))u(t-d_2) \\ +(B_\omega(r_t) + \Delta B_\omega(r_t))\omega(t) \\ x(t) = \varphi(t), t \in [-\tau, 0] \end{cases}, \tag{1}$$

where Ω is the sample space, F denotes the σ algebra subset on the sample space, and P indicates the probability density. $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ refers to the control input, $z(t) \in R^q$ is the control output and $\omega(t) \in R^p$ is an external interference input vector in $w(t) \in L_2[0,\infty)$. $L_2[0,\infty)$ is the square integrable function space, $\varphi(t)$ signifies a continuous initial state and $\{r_t, t\geq 0\}$ is a continuous time state of Markov process with the values in the finite space $\Lambda=\{1,2,\cdots,N\}$. The state transition probability is

$$p(r_{t+h}=j \mid r_t = i) = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1+\pi_{ij}h + o(h), & i = j \end{cases}, \tag{2}$$

where $h > 0$ and $\lim_{h\to 0} = o(h)/h = 0$. π_{ij} is the state transition probability from state i at time t to state j at time $t+h$. If $j \neq i$, then $\pi_{ij} > 0$. Otherwise, $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $B_d(r_t)$ and $B_\omega(r_t)$ are known constant real matrices of appropriate dimensions; $\Delta A(r_t)$, $\Delta A_d(r_t)$, $\Delta B(r_t)$, $\Delta B_d(r_t)$ and $\Delta B_\omega(r_t)$ are time-varying parameter uncertainties which satisfy the following condition

$$\begin{aligned} &\begin{bmatrix}\Delta A(r_t) & \Delta A_d(r_t) & \Delta B(r_t) & \Delta B_d(r_t) & \Delta B_\omega(r_t)\end{bmatrix} \\ &= E(r_t)F(t)[H_1(r_t) \quad H_2(r_t) \quad H_3(r_t) \quad H_4(r_t) \quad H_5(r_t)], \end{aligned} \tag{3}$$

where $E(r_t)$ and $H_i(r_t)(i = 1,2,3,4,5)$ are known constant real matrices of appropriate dimensions. $F(t)$ denotes an unknown matrix function with measurable elements and satisfies $F^T(t)F(t) \leq I$, I is unit matrices. d_1 and d_2 are time-delay parameters, which satisfy $d_1 > 0$, $d_2 > 0$, $\tau = \max\{d_1, d_2\}$.

In this paper, we will design a feedback controller

$$u(t) = K_i x(t), \tag{4}$$

where $K_1 \in R^{m\times n}$ is a constant matrix with appropriate dimension.

The stochastic Markov jump time-delay closed-loop system is as follows:

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{A}_d x(t-d_1) \\ \qquad + \bar{B}_d x(t-d_2) + \bar{B}_\omega \omega(t), \\ x(t) = \varphi(t), t \in [-\tau, 0], \end{cases} \tag{5}$$

where

$$\begin{aligned} &\bar{A} = A + \Delta A + \bar{B}, \bar{B} = (B + \Delta B)K, \\ &\bar{A}_d = A_d + \Delta A_d, \bar{B}_d = (B_d + \Delta B_d)K, \\ &\bar{B}_\omega = B_\omega + \Delta B_\omega. \end{aligned}$$

Definition 1. [30] *In the stochastic Markov jump time-delay system (1), when $u(t) = 0$ and $\omega(t) = 0$, if for all initial states x_0, initial mode r_0, system uncertainties (3) and all finite functions $\varphi(t)$ defined in $[-\tau, 0]$, we have*

$$E\{\int_0^\infty \| x(t,\varphi(t), r_0)\|^2 dt\} < \infty, \tag{6}$$

then the stochastic Markov jump time-delay system (1) is asymptotically stable.

Definition 2. [23] *Given a scalar $\gamma > 0$, if there is a state feedback controller (4) such that for all possible actuator failures, any $x_0 \in R^n$, $r_0 \in \Delta$ and system uncertainties (3), the closed-loop system is asymptotically stable and satisfies*

$$E\{\int_0^\infty [z^T(t)z(t)dt\} \leq \gamma^2 [\int_0^\infty \omega^T(t)\omega(t)]dt], \tag{7}$$

then the stochastic Markov jump time-delay system (1) is asymptotically stable and has the H_∞ performance index $\gamma > 0$. The corresponding controller is the robust H_∞ fault-tolerant controller.

Lemma 1. [31] *(Schur Complements). Given the symmetric matrix*

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix},$$

where Θ_{11} and Θ_{21} *are symmetric matrices, the following conditions are equivalent*:

$$\begin{aligned} &(I)\ \Theta < 0, \\ &(II)\ \Theta_{11} < 0, \Theta_{22} - \Theta_{21}\Theta_{11}^{-1}\Theta_{12} < 0, \\ &(III)\ \Theta_{22} < 0, \Theta_{11} - \Theta_{12}\Theta_{22}^{-1}\Theta_{21} < 0. \end{aligned}$$

Lemma 2. [32] *For the uncertainty $F(t)$ and the matrices $M = M^T$, S* and *N with appropriate dimensions, the following two conditions are equivalent:*

（1） $M + SF(t)N + N^T F^T(t) S^T < 0$,

（2） *existing* $\rho > 0$,

$$\begin{bmatrix} M & \rho S & N^T \\ \rho S^T & -\rho I & \rho J^T \\ N & \rho J & -\rho I \end{bmatrix} < 0.$$

3. ROBUST H∞ FAULT-TOLERANT CONTROL WITH THE ACTUATOR FAULT SYSTEM

In the system, the following actuator failure is introduced:

$$\begin{aligned} u^F(t) &= \left[u_1^F(t), \ldots, u_m^F(t)\right] \\ &= \left[I - \rho(t)\right]u(t), \end{aligned} \tag{8}$$

where $\rho(t) = diag\{\rho_1(t), \cdots \rho_m(t)\}$, $\rho_i(t)$ represents an unknown actuator failure factor, $\bar{\rho}_i$ and $\underline{\rho}_i$ are the upper and lower bounds of the actuator failure factor ρ_i.

According to the operation of the actuator, there is $0 \le \underline{\rho_i} \le \rho_i \le \overline{\rho_i} \le 1$. When $\underline{\rho_i} = \overline{\rho_i} = 0$, the i_{th} actuator works properly; when $\underline{\rho_i} = \overline{\rho_i} = 1$, the i_{th} actuator has a failure; when $0 \le \underline{\rho_i} \le \overline{\rho_i} < 1$, the i_{th} actuator has a partial failure.

When an actuator failure occurs in a Markov time-delay system, the state feedback controller is

$$u(t) = (I - \rho(t))K_i x(t) = L_i K_i x(t). \tag{9}$$

Bringing the equation (9) into the equation (5), the actuator fault closed-loop system of the stochastic Markov jump time-delay systems is

$$\begin{cases} \dot{x}(t) = \overline{A}x(t) + \overline{A}_d x(t-d_1) + \overline{B}_d x(t-d_2) \\ \qquad +\overline{B}_\omega \omega(t), \\ x(t) = \varphi(t), t \in [-\tau, 0], \end{cases} \tag{10}$$

where

$$\overline{A} = A + \Delta A + \overline{B}, \overline{B} = (B + \Delta B)LK,$$
$$\overline{A}_d = A_d + \Delta A_d, \overline{B}_d = (B_d + \Delta B_d)LK,$$
$$\overline{B}_\omega = B_\omega + \Delta B_\omega.$$

Theorem 1. *Consider the closed-loop system (10) with actuator failures, if there exist symmetric positive definite symmetric matrices P_i, Q_i, $R_i \in R^{n\times n}$ and matrix $K_i \in R^{m\times n}$, and the constant $\gamma > 0$ such that the following matrix inequality (11) holds*

$$\begin{bmatrix} \Omega_{11} & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ \Omega_{31} & 0 & \Omega_{33} & * \\ \Omega_{41} & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{11}$$

where

$$\Omega_{11} = \overline{A}_i^T P_i + P_i^T \overline{A}_i + Q_i + R_i + \sum_{j=1}^{N} \pi_{ij} P_j,$$
$$\Omega_{21} = \overline{A}_{di}^T P_i, \Omega_{31} = \overline{B}_{di}^T P_i,$$
$$\Omega_{22} = -Q_i, \Omega_{33} = -R_i, \Omega_{41} = \overline{B}_{\omega i}^T P_i.$$

Then the system (10) is asymptotically stable and can meet the H_∞ performance index $\gamma > 0$.

Proof. Let $\omega(t) = 0$, construct a Lyapunov functional candidate as

$$V(x_t, r_t, t) = \sum_{i=1}^{3} V_i(x_t, r_t, t), \tag{12}$$

where

$$V_1(x_t, r_t, t) = x^T(t) P_i x(t),$$
$$V_2(x_t, r_t, t) = \int_{t-d_1}^{t} x^T(s) Q_i x(s) ds,$$
$$V_3(x_t, r_t, t) = \int_{t-d_2}^{t} x^T(s) R_i x(s) ds.$$

In Euclidean space, the weak infinitesimal operator for the Lyapunov function with the Markov jump process is defined as follows:

$$\begin{aligned} &\ell V(x_t, r_t, t) \\ &= \lim_{\Delta t \to 0} \frac{1}{\Delta t}[E\{V(x_{t+\Delta t}, r_{t+\Delta t}, t+\Delta t)\} - V(x_t, r_t, t)] \\ &= \frac{\partial}{\partial t} V(x_t, r_t, t) + \frac{\partial}{\partial t} V(x_t, r_t, t)\dot{x}(r_t) \\ &\quad + \sum_{j=1}^{N} \pi_{ij} V(x_t, r_t, t). \end{aligned}$$

Then the derivative of $V(x_t, r_t, t)$ for time along the closed-loop system (10) is as follows:

$$\ell V(x_t, r_t, t) = \sum_{i=1}^{3} \ell V_i(x_t, r_t, t), \tag{13}$$

where

$$\begin{aligned} \ell V_1(x_t, r_t, t) &= \dot{x}^T(t) P_i x(t) + x^T(t) P^T{}_i \dot{x}(t) \\ &\quad + \sum_{j=1}^{N} \pi_{ij} x^T(t) P_j x(t), \\ \ell V_2(x_t, r_t, t) &= x^T(t) Q_i x(t) \\ &\quad - x^T(t-d_1) Q_i x(t-d_1), \\ \ell V_3(x_t, r_t, t) &= x^T(t) R_i x(t) \\ &\quad - x^T(t-d_2) R_i x(t-d_2). \end{aligned}$$

Substituting the first formula in the equation (10) into (13), we have

$$\begin{aligned} \ell V(x_t, r_t, t) &= \dot{x}^T(t) P_i x(t) + x^T(t) P_i^T \dot{x}(t) \\ &\quad + \sum_{j=1}^{N} \pi_{ij} x^T(t) P_j x(t) + x^T(t) Q_i x(t) \\ &\quad - x^T(t-d_1) Q_i x(t-d_1) + x^T(t) R_i x(t) \\ &\quad - x^T(t-d_2) R_i x(t-d_2) \\ &= (\overline{A}_i x(t) + \overline{A}_{di} x(t-d_1) \\ &\quad + \overline{B}_{di} x(t-d_2))^T P_i x(t) + x^T(t) P^T{}_i (\overline{A}_i x(t) \\ &\quad + \overline{A}_{di} x(t-d_1) + \overline{B}_{di} x(t-d_2)) \\ &\quad + \sum_{j=1}^{N} \pi_{ij} x^T(t) P_j x(t) + x^T(t) Q_i x(t) \\ &\quad - x^T(t-d_1) Q_i x(t-d_1) + x^T(t) R_i x(t) \\ &\quad - x^T(t-d_2) R_i x(t-d_2) = \xi^T(t) \Omega_1 \xi(t), \end{aligned}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & * & * \\ \Omega_{21} & \Omega_{22} & * \\ \Omega_{31} & 0 & \Omega_{33} \end{bmatrix}, \xi(t) = \begin{bmatrix} x(t) \\ x(t-d_1) \\ x(t-d_2) \end{bmatrix}.$$

From the equation (11) and Lemma 1, $\Omega_1 < 0$ can be derived, i.e. $\ell V(x_t, r_t, t) \le 0$. According to Definition 1, the closed-loop system (10) is asymptotically stable.

Below, the closed-loop system (10) is discussed with the H_∞ performance index γ. Under zero initial conditions, for any non-zero external perturbations $\omega(t) \in L^2[0,\infty]$, the derivative of $V(x_t, r_t, t)$ along the closed-loop system (10) is

$$\begin{aligned}\ell V_\omega(x_t, r_t, t) &\le (\bar{A}_i x(t) + \bar{A}_{di} x(t-d_1)\\ &+\bar{B}_{di} x(t-d_2) + \bar{B}_{\omega i}\omega(t))^T P_i x(t)\\ &+x^T(t) P_i^T (\bar{A}_i x(t) + \bar{A}_{di} x(t-d_1)\\ &+\bar{B}_{di} x(t-d_2) + \bar{B}_{\omega i}\omega(t)) + x^T(t) Q_i x(t)\\ &+\sum_{j=1}^{N} \pi_{ij} x^T(t) P_j x(t) - x^T(t-d_1) Q_i x(t-d_1)\\ &+x^T(t) R_i x(t) - x^T(t-d_2) R_i x(t-d_2),\end{aligned}$$

then

$$\begin{aligned}&z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t) + \ell V_\omega(x_t, r_t, t)\\ &= \varsigma^T(t)\Omega_2\varsigma(t),\end{aligned}$$

where

$$\Omega_2 = \begin{bmatrix} \Omega_{11} & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ \Omega_{31} & 0 & \Omega_{33} & * \\ \Omega_{41} & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\varsigma(t) = \begin{bmatrix} x(t) \\ x(t-d_1 \\ x(t-d_2 \\ \omega(t) \end{bmatrix},$$

$\Omega_1 < 0$ can be obtained from (11), which is $z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \ell V_\omega(x_t, r_t, t) < 0$.

Then the equation (14) can be derived by Dynkin's formula

$$\begin{aligned}&E\{\int_0^\infty [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)]dt\}\\ &+E\{V_\omega(x_t, r_t, t)\} - E\{V_\omega(x_0, r_0, t_0)\} < 0, \qquad (14)\end{aligned}$$

where x_0, r_0 and t_0 are the initial values of the corresponding variables. From the equation (14), the following can be obtained:

$$E\{\int_0^\infty [z^T(t)z(t)dt\} \le \gamma^2 [\int_0^\infty \omega^T(t)\omega(t)]dt]. \qquad (15)$$

As can be seen from Definition 2, the actuator fault closed-loop system (10) is stable and can meet the H_∞ performance index.

According to Theorem 1, the algorithm for the controller solving is given below.

Theorem 2. *For stochastic Markov jump time-delay systems with actuator failures (10), if there exist matrices and $X_i > 0$, $\hat{P}_i > 0$, $\hat{Q}_i > 0$, $\hat{R}_i > 0$ and K_i, $W_i \in R^{m\times n}$* and *$\gamma > 0$ such that the following matrix inequality holds, then we obtain*

$$\begin{bmatrix} \hat{\Omega}_{11} & * & * & * & * & * \\ \hat{\Omega}_{21} & \hat{\Omega}_{22} & * & * & * & * \\ \hat{\Omega}_{31} & 0 & \hat{\Omega}_{33} & * & * & * \\ \hat{\Omega}_{41} & 0 & 0 & -\gamma^2 I & * & * \\ \rho E^T & 0 & 0 & 0 & -\rho I & * \\ \hat{\Omega}_{61} & H_2 X_i & H_4 L_i W_i & H_5 & \rho J & -\rho I \end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned}\hat{\Omega}_{11} &= (A_i X_i + B_i L_i W_i)^T + (A_i X_i + B_i L_i W_i)\\ &\quad +\hat{Q}_i + \hat{R}_i + \sum_{j=1}^{N} \pi_{ij} P_j X_i, \hat{\Omega}_{21} = X_i A_{di}^T,\\ \hat{\Omega}_{31} &= (B_{di} L_i W_i)^T, \hat{\Omega}_{22} = -\hat{Q}_i, \hat{\Omega}_{33} = -\hat{R}_i,\\ \hat{\Omega}_{41} &= B_{\omega i}^T, \hat{\Omega}_{61} = H_1 X_i + H_3 L_i W_i.\end{aligned}$$

Then $K_i = W_i X_i^{-1}$ is the robust H_∞ fault tolerant controller of the closed-loop system (10).

Proof. Let $P_i = X_i^{-1}$, $K_i = W_i X_i^{-1}$, $Q_i = X_i^{-1}\hat{Q}_i X_i^{-1}$, $R_i = X_i^{-1}\hat{R}_i X_i^{-1}$ pre- and post-multiplying both sides of (16) by $\{X_i^{-1}\ X_i^{-1}\ X_i^{-1}\ X_i^{-1}\ I \cdots I\}$. From Lemma 2, the equation (16) is equivalent to the equation (11). The proof is completed.

The condition for the asymptotic stability of the closed-loop system under actuator failure is given by Theorem 1. The solution of the robust H∞ fault-tolerant controller is provided by Theorem 2. The conclusions address the stochastic Markov jump time-delay system with parameter uncertainties. Compared with the existing references, the proposed fault-tolerant control does not need to estimate the boundary value of the actuator failure.

4. EXPERIMENTAL SIMULATION

4.1. Numerical simulation

Consider the stochastic Markov jump time-delay systems with the following parameters,
mode 1:

$$A = \begin{bmatrix} 0.8 & 0.5 \\ -0.5 & 0.3 \end{bmatrix}, A_d = \begin{bmatrix} 0.1 & 0.5 \\ 0.2 & -0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1 & 0.8 \\ 0.5 & 0.6 \end{bmatrix}, B_d = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix},$$

$$B_\varpi = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

mode 2:

$$A=\begin{bmatrix}0.5 & 0.2\\ -0.1 & 0.5\end{bmatrix}, A_d=\begin{bmatrix}0.2 & 0.1\\ 0 & 0.2\end{bmatrix},$$

$$B=\begin{bmatrix}0.2 & 0.2\\ 0.3 & 0.6\end{bmatrix}, B_d=\begin{bmatrix}0.1 & 0.1\\ 0.3 & 0.2\end{bmatrix},$$

$$B_\varpi=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix},$$

where

$$E=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, H_1=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$

$$H_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}, H_3=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix},$$

$$H_4=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix}, H_5=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$

$$J=0, F(t)=\begin{bmatrix}\sin(t) & 0\\ 0 & \cos(t)\end{bmatrix},$$

$$\omega(t)=\begin{bmatrix}e^{-t}*\cos(t)\\ e^{-t}*\sin(t)\end{bmatrix}.$$

The transition probability matrix is $\pi_{ij}=\begin{bmatrix}-2 & 2\\ 1 & -1\end{bmatrix}$, $d_1=d_2=0.5$, $\lambda=0.2$, $\rho=0.5$.

Consider a system actuator failure, $L_0=diag(1, 1)$ is actuator normal, $L_1=diag(0, 1)$ and $L_2=diag(1, 0)$ indicate an actuator failure. When the second channel faults occur in the system, $\gamma=0.9828$ is obtained by using the MATLAB LMI toolbox. The state feedback gain matrices are as follows:

$$K_1=\begin{bmatrix}-6.4338 & -0.2554\\ -0.2554 & -6.6371\end{bmatrix},$$

$$K_2=\begin{bmatrix}-10.6675 & -0.2927\\ -0.2927 & -11.3763\end{bmatrix}.$$

The state response curves with the actuator 2 failure are shown in Figs 1 and 2.

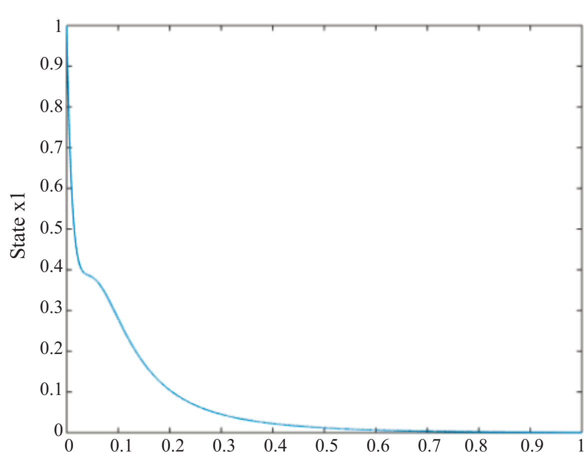

Fig. 1. The x1 status with the actuator 2 failure.

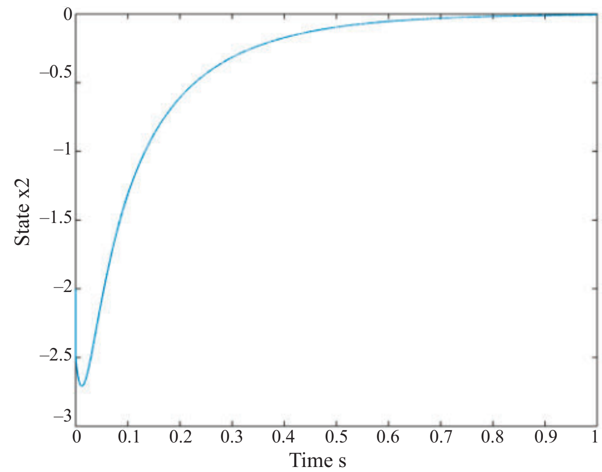

Fig. 2. The x2 status with the actuator 2 failure.

When the first channel faults occur in the system, $\gamma=1.1373$ is obtained by using the MATLAB LMI toolbox. The state feedback gain matrices are as follows:

$$K_1=\begin{bmatrix}-4.0525 & 0.2060\\ 0.2060 & -4.1882\end{bmatrix},$$

$$K_2=\begin{bmatrix}-7.9843 & -0.1906\\ -0.1906 & -7.9945\end{bmatrix}.$$

At this time, the state response curves with actuator 1 failure are shown in Figs 3 and 4.

In the case of an actuator failure, it can be seen from the simulation results that the designed controller can ensure that the system has certain anti-interference and fault tolerance. This verifies the effectiveness of the proposed method.

4.2. UAV application simulation

To further verify the effectiveness of the proposed method, the robust fault-tolerant control method is applied to the

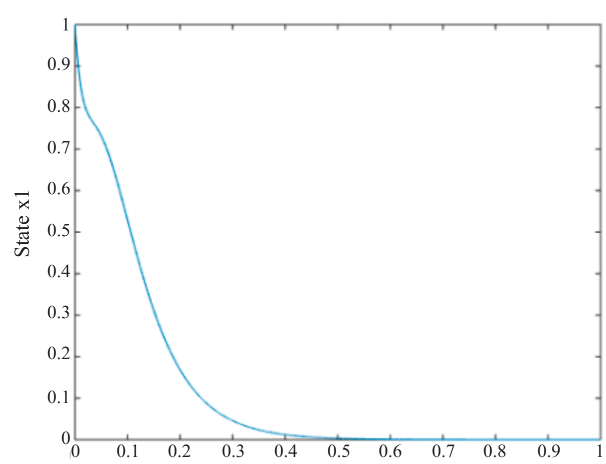

Fig. 3. The x1 status with the actuator 1 failure.

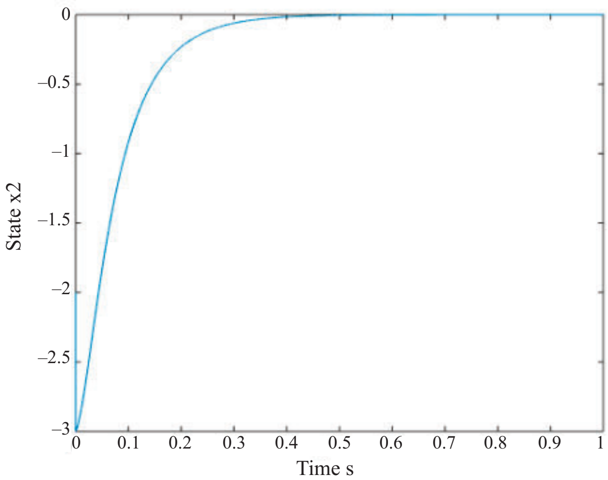

Fig. 4. The x2 status with the actuator 1 failure.

quad-rotor UAV control system. With reference to [33], a linearized four-rotor UAV model is selected, and its parameter matrices are as follows:

$$A(t) = \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643+0.5\beta(t) & -0.4+\beta(t) & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.2 & 0 & 0.3 \\ 0.1 & 0.5 & 0 \\ 0 & 0.1 & 0.2 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0 & 0.05 \\ 0.03 & 0.2 & 0 \\ 0 & 0.05 & 0.1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \end{bmatrix}, E_2 = \begin{bmatrix} -0.1 \\ 0 \\ -0.2 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, F_1 = F_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

where $\beta(t)$ is an uncertain model parameter that satisfies the Markov jumping process of the mode

$N = 2$:

$$\beta(t) = \begin{cases} -1, r(t) = 1 \\ -2, r(t) = 2. \end{cases}$$

Considering that the first failure occurs in the system, the other parameters selected are the same as the numerical example. $x(t) = [x_1\, x_2\, x_3]^T = [\theta\ \phi\ \psi]^T$ is the state vector, where ϕ is roll angle, θ is pitch angle, and ψ is yaw angle.

In order to apply simulation, the initial state is set to $x_0(1,\ 0.1\ 0.1)$, the actuator failure is set to $f_a(t) = \sin(t)$. $\gamma = 4.4577$ is obtained by using the MATLAB LMI toolbox. The state feedback gain matrices are as follows:

$$K_1 = [-90.5965 \quad 3.2325 \quad -27.1382],$$
$$K_2 = [39.7603 \quad 4.1709 \quad 35.10].$$

The roll angle, pitch angle, and yaw angle curves are shown in Figs 5–7. It can be seen from Figs 5–7 that there is a certain transition process in the initial operating state of the system. However, under the action of the fault-tolerant controller, the roll angle, pitch angle, and yaw angle of the UAV can reach stability after a short adjustment process. The results show that the designed

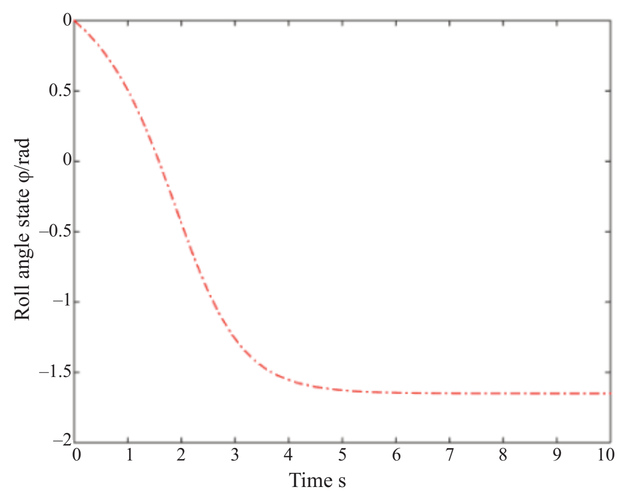

Fig. 5. The curve of roll angle ϕ.

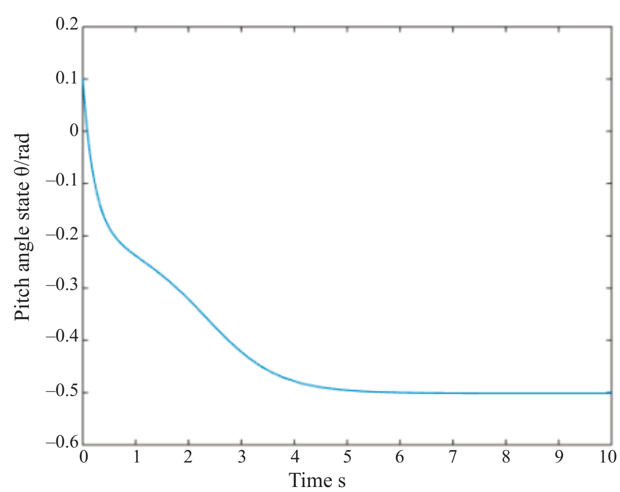

Fig. 6. The curve of pitch angle θ.

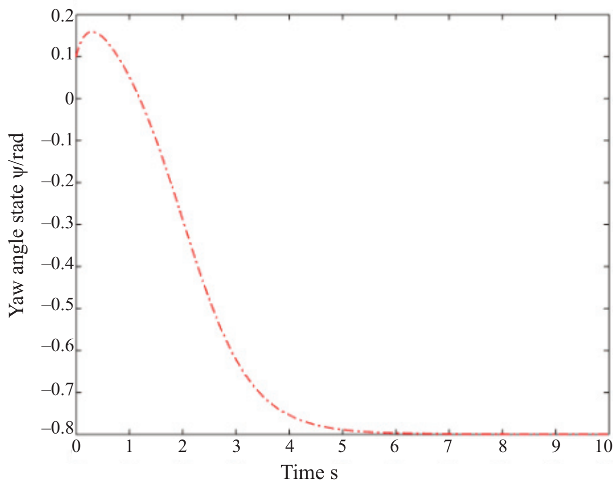

Fig. 7. The curve of yaw angle ψ.

fault-tolerant controller has a good control effect when it is used in a four-rotor UAV system. The effectiveness of the proposed method is verified.

5. CONCLUSIONS

In this paper, the robust H∞ fault-tolerant control for uncertain stochastic Markov jump systems with both state time-delay and input time-delay has been studied. Based on the linear matrix inequality, by constructing the Lyapunov functional, the sufficient condition is presented for the asymptotic stability of the closed-loop system under actuator failure. Moreover, the solution of the fault-tolerant controller is also provided, so that the closed-loop system satisfies a certain H∞ suppression index γ. The validity of the method is verified by the numerical simulation examples and UAV application simulation.

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Vigase täiturmehhanismiga stohhastiliste ajalise hilistumisega Markovi hüppesüsteemide robustne H∞ veakindel juhtimine ja rakendamine

Fu Xingjian ja Pang Xinrui

On uuritud robustset H∞-meetodil põhinevat veakindlat juhtimist parameetriliselt ebatäpsete stohhastiliste ajalise hilistumisega Markovi hüppesüsteemide jaoks. Lyapunovi stabiilsusteooria abil on leitud piisav tingimus lineaarse maatriksvõrratuse kujul robustse kontrolleri olemasoluks, mis garanteerib täiturmehhanismi rikke korral suletud süsteemi asümptootilise stabiilsuse ja etteantud tulemuslikkuse. Robustne veakindel juhtimisalgoritm on leitud lineaarse maatriksvõrratuse lahendina. Juhtimisalgoritmil on lihtne struktuur ja selle leidmine ei nõua palju arvutusi. Meetodi kehtivust on kontrollitud akadeemilise näite ja mehitamata õhusõiduki mudeli numbriliste simulatsioonide kaudu.