

<https://doi.org/10.3176/oil.2001.3.03>

THE EFFECT OF SUBSURFACE MINING ON THE LAND SURFACE AND PRELIMINARY CALCULATION OF PILLARS

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In Estonia oil shale is produced by underground and surface mining. The technologies used exert a substantially different effect on the topography and water regime. The cavities generated by subsurface mining may lead to a series of deformations that travel to the land surface and hamper further use of mined-out areas. Based on mathematical calculations, prognostication of ground subsidence in dependence of pillar sizes is attempted.

Introduction

Mining processing and utilization of oil shale in Estonia has led to serious environmental consequences. The latter exert influence far behind the boundaries of the mining area and long after the cessation of mining.

The first oil shale mines before World War II were located in areas where it was either exposed on the ground or occurred relatively close to the surface and therefore mainly open-cast mining was used. Underground mining in earlier times was introduced already in areas where the thickness of overburden exceeded 5–8 m, now it starts from several tens of metres. The chambers generated by subsurface mining cause deformations that reach up to the ground surface and hamper further use of the mined-out areas. Due to that, and taking into account great losses in underground mining in the Estonian National Environmental Strategy [1], the open-cast mining is recommended as much as possible. But where it is impossible, underground mining should be used.

In Estonian underground mines oil shale has been produced by pillar-and-double-stall mining, double-face-longwall-stall mining, fully mechanized narrow-web mining in the longwall-stall system and room-and-pillar mining.

The effect of mining on the topography differs with mining technologies and has been dealt with in many recent publications, in most detail by Toomik [2]. In all cases, the mining plot is of a rectangular shape. The present paper focuses on the study of the processes acting during underground mining. Their dependence on the size of faces is analysed. Long-term stability of the ground is estimated, with special attention on potential after-deformations. This kind of knowledge is important from the standpoint of land-use planning.

Rate of Vertical and Horizontal Deformation

The rates of deformation have been calculated according to Laigna and Joosep [3]. The basic notions used are:

1. Subsidence η . Subsidence (absolute vertical displacement) is the value characterising the displacement of a point of the land surface in the vertical direction.
2. Inclination i is the difference of the ratio of the subsidence of two points versus the distance between these points
3. The curvature of trough shift k is the difference ratio of the dip of two neighbouring trough intervals versus the half-sum of the length of these intervals

$$\frac{i}{k} = R_k - \text{curve radius}$$

4. Horizontal shift ξ is a horizontal assembly of points of displacement vectors in the trough.
5. Horizontal deformations Σ are extensions or compressions caused by non-uniform shift of points in the horizontal plane. They represent the difference of the ratio of the lengths of intervals in the horizontal plane versus its initial length.

Determining the rates of deformations in solving the planar task, an initial model for the calculation of the land surface, expressed by the Gaussian function, was obtained:

$$\eta(x, y) = -\frac{\eta_0}{2} \left[\Phi \left(\frac{2D_p - x}{cy} \right) + \Phi \left(\frac{x}{cy} \right) \right] \quad (1)$$

with boundary conditions

$$-\infty < x < 0, \eta = 0$$

$$2D_p < x < \infty, \eta = 0$$

$$0 \leq x \leq 2D_p, \eta = -\eta$$

where $2D_p$ is the length of the face;

$$C = \frac{C_2}{C_1}; C_2 = \frac{E}{6(1+\nu)}, C_1 = -\frac{\sigma}{\eta};$$

where C_1 is coefficient of the resistance of soil to compression;

σ is intensity of vertical removal of soil;

E is modulus of soil elasticity (Young's modulus);

ν is Poisson's coefficient.

For the ground surface points $y = H$, we obtain instead (1):

$$\eta(x, H) = -\frac{\eta_0}{2} \left[\Phi \left(\frac{2D_p - x}{cH} \right) + \Phi \left(\frac{x}{cH} \right) \right] \quad (2)$$

The rate of vertical subsidence due to face displacement:

$$\frac{d\eta}{dt} = \frac{\partial \eta}{\partial D_p} \cdot \frac{dD_p}{dt}$$

The rate of face movement

$$v = \frac{d}{dt} 2D_p = 2 \frac{dD_p}{dt} \quad (3)$$

The rate of vertical subsidence

$$u = \frac{d\eta}{dt} \quad (4)$$

Differentiating Equation (1.2) we obtain

$$u = -\frac{v}{2c} \cdot \frac{\eta_0}{H} \Phi' \left(\frac{2D_p - x}{cH} \right) \quad (5)$$

The maximum rate of sinking is in the point with the coordinate

$$X = 2D_p, \quad (6)$$

i.e. in the completely mined-out area the maximum rate of subsidence occurs in the point of the trough bend displacement at the distance $a/2$ from the boundary of the working (Fig. 1).

Inserting (6) into (5) the maximum rate of subsidence is

$$u_{\max} = -v \frac{\eta_0}{H} \cdot \frac{1}{c\sqrt{2\pi}} \quad (7)$$

Figure 1 shows the distribution of rates along displacement trough.

The value of the maximum rate of vertical subsidence and location of the point with the maximum rate with respect to the face does not depend on the distance of the face from the excavation, which is confirmed by *in situ* observations.

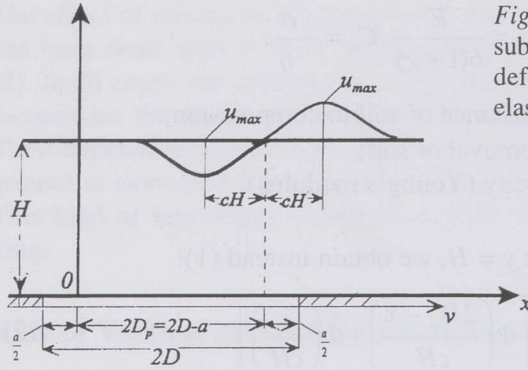


Fig. 1. Distribution of rates of vertical subsidence and absolute horizontal deformations (a is the width of the elastic bending of the immediate roof)

The rate of absolute deformations may be dealt with in the same way. Differentiating Formula (2), we find the dip of displacement trough:

$$\frac{d\eta}{dx} = \frac{\eta_0}{2cH} \left[\Phi' \left(\frac{2D_p - x}{cH} \right) - \Phi' \left(\frac{x}{cH} \right) \right] \quad (8)$$

and also absolute horizontal deformations:

$$\xi(x) = -\frac{\eta_0}{2} c \left[\Phi \left(\frac{2D_p - x}{cH} \right) - \Phi \left(\frac{x}{cH} \right) \right] \quad (9)$$

Let us denote the rate of absolute horizontal deformations through

$$w = \frac{d\xi}{dt} = \frac{d\xi}{dD_p} \cdot \frac{dD_p}{dt} \quad (10)$$

Differentiating Equation (9)

$$w = v \frac{\eta_0}{cH^2} \cdot \frac{1}{2} (2D_p - x) \cdot \Phi' \left(\frac{2D_p - x}{cH} \right) \quad (11)$$

Differentiating Equation (11) in terms of x and equalizing the derivative to zero, we can find the place and position of the maximums of absolute horizontal deformation rates:

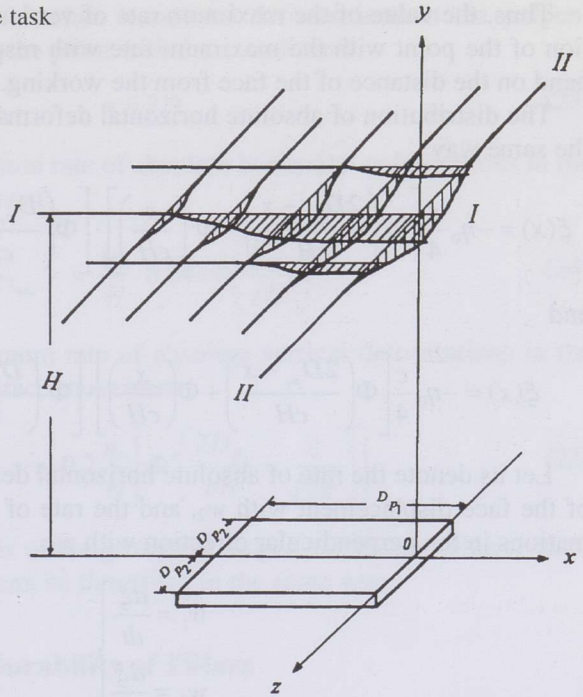
$$x = 2D_p \pm cH \quad (12)$$

The maximum value of the absolute horizontal deformation rate is determined inserting (12) into (11)

$$w_{\max} = \pm v \cdot 0.242 \frac{\eta_0}{H} \quad (13)$$

The distribution of rates of the distribution of inclinations, curvature, relative horizontal deformations, etc. may be studied in the same way. The volume task look at (Fig. 2).

Fig. 2. Scheme of the volume task



Let us take the beginning of the working as an initial point for the axis of coordinates headed in the direction of the face displacement. Making calculations in the same way as in solving the planar task, the values of vertical sinkings on the ground surface are found according to the following equation:

$$\eta(x, z, H) = -\frac{\eta_0}{4} \left[\Phi \left(\frac{2D_{p1} - x}{cH} \right) + \Phi \left(\frac{x}{cH} \right) \right] \cdot \left[\Phi \left(\frac{D_{p2} + z}{cH} \right) + \Phi \left(\frac{D_{p2} - z}{cH} \right) \right] \quad (14)$$

In that case, the rate of vertical subsidence is expressed as follows:

$$u = -v \frac{\eta_0}{H} \cdot \frac{1}{4c} \Phi' \left(\frac{2D_{p1} - x}{cH} \right) \left[\Phi \left(\frac{D_{p2} + z}{cH} \right) + \Phi \left(\frac{D_{p2} - z}{cH} \right) \right] \quad (15)$$

Differentiating x and z and equalizing the derivatives to zero, we find that the maximum rate of sinking occurs in the point with the coordinates

$$x = 2D_{p1}, z = 0, \quad (16)$$

the value of the maximum subsidence rate is

$$u_{\max} = -v \frac{\eta_0}{H} \Phi \left(\frac{D_{p2}}{cH} \right) \cdot \frac{1}{c\sqrt{2\pi}} \quad (17)$$

Thus, the value of the maximum rate of vertical subsidence and the position of the point with the maximum rate with respect to the face do not depend on the distance of the face from the working.

The distribution of absolute horizontal deformations may be calculated in the same way:

$$\xi(x) = -\eta_0 \frac{c}{4} \left[\Phi' \left(\frac{2D_{p_1} - x}{cH} \right) - \Phi' \left(\frac{x}{cH} \right) \right] \cdot \left[\Phi \left(\frac{D_{p_2} + z}{cH} \right) + \Phi \left(\frac{D_{p_2} - z}{cH} \right) \right], \quad (18)$$

and

$$\xi(x) = -\eta_0 \frac{c}{4} \left[\Phi \left(\frac{2D_{p_1} - x}{cH} \right) + \Phi \left(\frac{x}{cH} \right) \right] \cdot \left[\Phi' \left(\frac{D_{p_2} + z}{cH} \right) + \Phi' \left(\frac{D_{p_2} - z}{cH} \right) \right]$$

Let us denote the rate of absolute horizontal deformations in the direction of the face displacement with w_2 , and the rate of absolute horizontal deformations in the perpendicular direction with w_1 :

$$\left. \begin{aligned} w_1 &= \frac{d\xi}{dt} \\ w_2 &= \frac{d\xi}{dt} \end{aligned} \right\} \quad (19)$$

Differentiating (18) in terms of time and marking, as earlier, the rate of the face shift with $v = \frac{d}{dt} 2D_{p_1}$:

$$w_1 = -v \cdot \frac{\eta_0}{H} \cdot \frac{1}{4} \Phi \left(\frac{2D_{p_1} - x}{cH} \right) \left[\Phi' \left(\frac{D_{p_2} + z}{cH} \right) - \Phi' \left(\frac{D_{p_2} - z}{cH} \right) \right], \quad (20)$$

and

$$w_2 = v \cdot \frac{1}{4} \cdot \frac{\eta_0}{cH^2} (2D_{p_1} - x) \Phi' \left(\frac{2D_{p_1} - x}{cH} \right) \left[\Phi \left(\frac{D_{p_2} + z}{cH} \right) + \Phi \left(\frac{D_{p_2} - z}{cH} \right) \right] \quad (21)$$

Differentiating (20) and (21) in terms of x and z and equalizing the derivatives to zero, we find that the maximum rate of absolute horizontal deformations in the direction of the face displacement $w_{2\max}$ is in the points with the coordinates

$$x = 2D_{p_1} \pm c \cdot H, \quad (22)$$

$$z = 0$$

where z is coordinate of the point where the trough bend intersects with the line drawn parallel to the face.

The maximum rate of absolute horizontal deformations in the perpendicular direction $w_{1_{\max}}$ is in the point with the coordinates

$$x = 2D_{p_1} \quad (23)$$

The value of the maximum rate of absolute horizontal deformations in the direction of face displacement

$$w_{2_{\max}} = v \cdot \frac{\eta_0}{H} \cdot 0.242 \Phi \left(\frac{D_{p_2}}{cH} \right) \quad (24)$$

The value of the maximum rate of absolute vertical deformations in the direction perpendicular to face movement

$$w_{1_{\max}} = v \cdot 0.2 \frac{\eta_0}{H} \left[\Phi' \left(\frac{2D_{p_2}}{cH} - \frac{2}{\sqrt{2\pi}} \right) \right] \quad (25)$$

The distribution of rates of the growth of slopes, curvature, relative horizontal deformations, etc., can be described in the same way.

Determination of the Durability of Pillars

Experimental studies in different countries with different rocks have shown that the strength of samples decreases with their size. In practical calculations on pillars a significant role is played by the critical value connected with the durability of rock massif. This characteristic is determined as the width of sample, starting from which its further increase does not cause any noteworthy decrease in durability.

The importance of the critical value effect lies in the circumstance that the values of durability limits determined on the samples with critical sizes can be applied immediately to real pillars.

Discrepancies between durability characteristics of small samples tested in laboratory conditions and those of real large pillars are characterized by an extent effect. Investigations showed that the characteristics of coal obtained through laboratory experiments with small samples, suit satisfactorily for *in situ* conditions if the following equation is used:

$$\sigma_1 = \frac{k}{\sqrt{36}} \quad (26)$$

for pillars of cubic shape more than 0.9 m in height, or with the equation

$$\sigma_1 = \frac{k}{\sqrt{h}} \quad (27)$$

for pillars of cubic shape less than 0.9 m in height.

In those two equations the constant for the material of which the real pillar consists must be determined from the equation

$$k = \sigma_c \sqrt{D} \quad (28)$$

where σ_c is the boundary limit of durability at one-axis pressure of rocks under laboratory conditions;

D is the length of the side of the sample if the sample is a cube, or diameter if the sample is a cylinder.

Of numerous different equations suggested for calculating the durability of pillars, the following ones have found a wider use. Based on laboratory experiments with samples of hard rocks and on the theory of durability, the following equation was obtained by Hustrulid:

$$\sigma_p = \sigma_1 (0.778 + 0.222w/h), \quad (29)$$

where σ_p is the boundary limit of the durability of pillar;

σ_1 is the boundary limit of the durability of a cubic sample at one-axis pressure ($w/h = 1$);

w and h are the width and height of the pillar, respectively.

Considering the conditions of the gravitational field of pressures this equation can be used if the ratio w/h has values from 0.25 to 4.0.

The following formula is also used:

$$\sigma_p = k \sqrt{\frac{w}{h}} \quad (30)$$

where k is the coefficient determined from Equation (28);

w and h are the measures of the pillar;

σ_p is the boundary of durability of pillar.

Another formula is suggested for calculating the durability of coal pillars

$$\sigma_p = \sigma_1 \sqrt{\frac{w}{h}} \quad (31)$$

where σ_1 is the boundary limit of durability of pillars of cubic shape ($w = h = 1$).

The state of several crushed and uncrushed pillars was studied in the Republic of South Africa. The following equation was used to determine the durability of pillars of square shape [4]:

$$\sigma_p = Kh^\alpha w^\beta \quad (32)$$

Constants (32) were determined from statistic analysis of data based on actual mining experience. Of the 125 pillars studied, 98 pillars continued being resistant to pressure and 27 crushed (crushing took place during the period of studies). In advancing an equation for calculating the durability of pillars it was assumed that the dimensions of the preserved pillars were large enough to keep them from crushing, while the width of the crushed pillars was too small. As a result, the following equation was obtained:

$$\sigma_p = 7.2w^{0.46} / h^{0.66} \quad (33)$$

where σ_p is expressed in Mpa and w and h in metres.

In using this equation it is recommended to insert the coefficient of safety equal to 1.6 with a possible range of change from 1.31 to 1.88.

Like all empirical equations, the suggested equations must be used within these diapasons of change in which they were derived.

Calculation of Pillars

Based on the above-described analysis the following method can be recommended for calculation of pillars in new pits where the room-and-pillar mining technique will be used.

1. On the basis of the known boundary limit of durability of the mineral resource at one-axial pressure, the value k is determined for the planned pillar. For that purpose, the following formula is used:

$$k = \sigma_c \sqrt{D} \quad (34)$$

where D is the diameter of cylindrical sample or the side length of the corresponding cubic sample.

2. An equation is chosen to calculate the width of the pillar at the known thickness of the bed h , for instance

$$\sigma_p = \sigma_1(0.64 + 0.36w/h) \quad (35)$$

where $\sigma_1 = k / \sqrt{36}$.

3. The pressure on the pillar (average pressure) is calculated on the basis of the region of roof:

$$S_p = 1.1H \left(\frac{w+B}{w} \right) \left(\frac{w+L}{L} \right) \quad (36)$$

where S_p is the pressure on the pillar;

H is depth from the daytime surface;

B is gap of the working;

w and L are width and length of the pillar, respectively.

In order to simplify calculations, it may be assumed that the pillar has a square cut ($w = L$), however, this will result in underestimation of pillar's durability.

4. Coefficient of safety F (usually 1.5–2.0) is chosen and the equation $\sigma_p/F = S_p$ is compiled which thereafter is solved in terms of w . To check the results of calculations a nomogram (Fig. 3) is used.

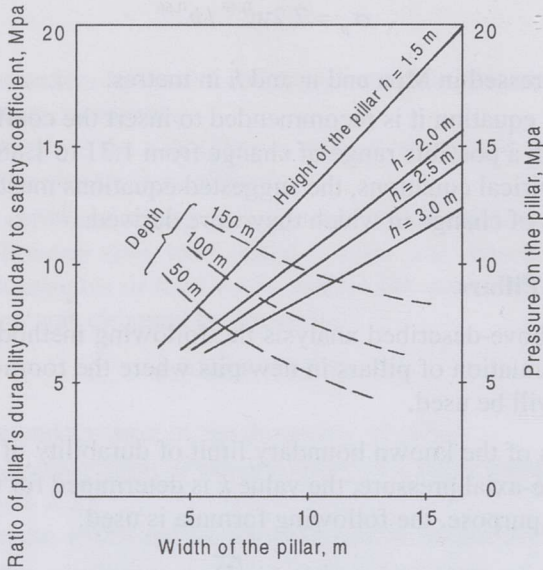


Fig. 3. Nomogram for determining the dimensions of pillars (oil-shale bed)

5. An economic assessment is carried out during which economically profitable mineral resources production will be determined. Optimal production is calculated from the equation

$$e = 1 - \left(\frac{w}{w + B} \right)^2 \tag{37}$$

6. If the production proves inadmissibly low and for its increase the width of the pillar must be reduced, then from the equation used in p.5, the width of the pillar is found, which guarantees the required equilibrium of the roof. It is also checked whether the durability conditions of the pillar are satisfactory. For that purpose the value of the coefficient of safety is calculated

$$F = \frac{\sigma_p}{S_p} \tag{38}$$

where σ_p is the boundary limit of durability of pillar, determined in p.2;
 S_p is the pressure on the pillar, determined in p.3.

The coefficient of safety must be within the range of 1.5–2.0.

- 7. The results obtained are checked using Equations (29), (30), and (33).
- 8. If the width of the chambers used does not exceed 20–30 m, the state of the land surface may stabilize after the subsidence of roof rocks has ceased.

Acknowledgements

The authors wish to thank Mrs. Helle Kukk for linguistic improvements and Ms Helle Pohl for typing the manuscript.

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Received February 14, 2001

Introduction

Recently a new test method for nitrite as nitrosoalkylresorcinolate was developed [1]. The method is based on the specifically easy nitrosation of alkylresorcinols (*R*) with nitrous acid formed from nitrite in acidic solutions, and on the subsequent formation of a coloured complex (MeA_n) with a transition metal cation (*Me*). The main oil shale originated phenols – 5-methylresorcinol (5-MR) and 2,5-dimethylresorcinol (2,5-DMR) – were applied as *R*. The cobalt and iron(II) cations were selected as the most appropriate *Me*. Experimental plots of optical density (*E*) versus pH and the initial concentrations of nitrite, *R* and *Me* (C_N , C_R and C_{Me}) were represented. The curves were obtained under the conditions where only one of the factors was varied and three of them were kept constant (optimal).

In this work, studies of MeA_n formation was described as a result of the joint effect of C_N , C_R and C_{Me} on *E*. To solve the problem, a mathematical model of the process was deduced governed by the physical-and-chemical laws being in force at equilibrium conditions. The molar extinction coeffi-