

## TEMPERATURE DEPENDENCY OF ELECTRICAL NETWORK LOAD

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*The dependency of electrical network load on outdoor temperature is examined in this paper. The mathematical model that considers the level of temperature dependency, inertia and non-linearity of the temperature influence, and changes with time is described.*

### Introduction

The dependency on outdoor temperature is one of the most important characteristics of load. First of all, this dependency is expressed there, where electrical heating or different air conditioning devices are used. For example, in Lapland (Finland), where electrical heating is of great importance and deviations in outdoor temperature are large, the load increase caused by outdoor temperature may be up to 100% compared to the load at normal temperature. Usually, however, the temperature dependency of load is smaller, especially in the case of industrial load.

The dependency of electrical network load on temperature and other weather factors is widely been covered in different papers. In [1] a regression-based adaptive weather-sensitive short-term load forecasting approach has been presented. Proposed algorithm is robust and not very sensitive to weather forecast accuracy considering only the data having a great influence on consumption, e.g. only minimal, maximal and average daily temperatures and average daily wind speed are used. Also the extent and inertia of the temperature influence are considered. Weather impact on load diversity is covered in [2]. To relate critical points in the load curve to weather factors, a multiple regression was applied. It is proposed that daily maximum temperatures are the dominant weather factors, followed by humidity at times of maximum temperature, daily minimum temperature, wind speed and hours of sunshine. When considering the temperature, the effect of the previous day temperature is important [3]. It is indicated that

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temperature of the previous day is more important than the peak day temperature. Physically that temperature represents the heat stored in buildings due to the previous day temperature.

Besides outdoor temperature, electrical network load depends on other meteorological factors such as sun radiation (cloudiness), wind speed, humidity, etc. To avoid unreasonable complexity of the model and possible estimation difficulties, we consider only the basic factor – outdoor temperature. It must be emphasised that for practicality, only factors that are treated (including forecasted) by meteorological services in a quantitative way should be considered. Principally other weather factors can be considered through transformed value of temperature, i.e. through effective temperature. Of course, this value must be also treated by meteorological services.

The mathematical model of the temperature dependency describes the increase in load when outdoor temperature changes. In addition to the level of load deviations due to the temperature, the delay of the temperature influence (inertia), changes in time and non-linearity must be considered. As known, the delay of about 24 hours is characteristic to the temperature influence. Yearly changes must be considered, since the temperature dependency in summer differs remarkably from that in winter. The variability of weekly and daily temperature dependencies must also be considered. A non-linearity phenomenon occurs when, in case of certain temperature values, the character of temperature dependency changes. For example, in summertime, when temperature dependency in northern lands is mostly missing, load increase can be recognised if temperature falls below 14 °C or rises above 25 °C. In wintertime, the speed of load increase may decrease if temperature falls below –25 °C. These phenomena can be explained by using additional heating or cooling equipment in summertime and achieving the maximum output of heating equipment or finishing outdoor works in wintertime.

The structure of the model is the same for all loads. In order to make the model to correspond to a certain load, model parameters must be estimated with that load data. Despite the modest number of parameters, some difficulties may arise when using formal estimation methods (e.g. least-square method). For example, in the summer period, when the temperature influence is small, bad differentiation between the delay of the temperature influence and the level of the temperature deviation causes problems. Furthermore, it is a problem to estimate the parameters that are associated with non-linearity. The fundamental problem during the observable period is the unilaterality of the weather type. Thus, for example, many years with warm winter, cold summer etc. may occur. The minimal period for the change of weather types from the meteorological point of view is 30 years. In the abovementioned range, the load data is either missing or is not reliable enough due to the changes of the load character. Therefore, it is necessary to estimate model parameters in two phases. In the first phase, during load research, larger amounts of loads are handled. As a result, the temperature

dependency models are identified – the type cases are found and some of the model parameters are fixed. In the second, model estimation phase, the rest of parameters are estimated, basing on the observable data of single loads.

### Mathematical model of the load

The mathematical model describing changes of load (active power, reactive power, or current) consists of three basic components [4]:

$$P(t) = E(t) + G(t) + T(t)$$

where  $E(t)$  is mathematical expectation of the load;  
 $G(t)$  is temperature-sensitive part of the load;  
 $T(t)$  is stochastic component of the load.

Mathematical expectation  $E(t)$  describes regular changes of a load, such as general trend and seasonal, weekly, and daily periodicity. Mathematical expectation is principally non-stochastic and corresponds to the normal temperature.

The temperature-sensitive part of a load  $G(t)$  describes load deviations caused by deviations of outdoor temperature from the normal temperature. The normal temperature (mathematical expectation of the temperature) is the average outdoor temperature of the last 30 years on any given hour of the year. If the real outdoor temperature corresponds to the normal temperature (considering delay), the value of the temperature-sensitive part of the load is zero. To compare temperature dependencies of different loads, the component  $G(t)$  is normalized

$$G(t) = R(t)\gamma(t),$$

where  $R(t)$  is rate of temperature dependency of the load;  
 $\gamma(t)$  is normalized component of temperature dependency.

The stochastic component  $T(t)$  describes stochastic deviations of load. Due to the autocorrelation, the deviations are stochastically dependent on each other. It is possible to describe the stochastic component of the load by expected deviation  $\zeta(t)$ , which represents the conditional mathematical expectation of the stochastic component and normally distributed non-correlated residual deviation (white noise)  $\xi(t)$ . In addition, it is necessary to consider peak deviations of the load by the component  $\pi(t)$ , which describes large positive or negative deviations that do not correspond to the normal distribution. It is practical to normalize the stochastic component. The proper rate is the standard deviation of the load  $S(t)$ . The result is

$$\Theta(t) = S(t)[\zeta(t) + \xi(t) + \pi(t)].$$

Let us observe the component of temperature dependency. Here, the rate of temperature dependency  $R(t)$  makes the relative components of temperature dependency  $\gamma(t)$  comparable for different loads. It enables us, for example, to

estimate the component  $\gamma(t)$  according to the load class simultaneously for many loads. The rate  $R(t)$  also determines the level of the temperature dependency for every single load and supports the consideration of its temporal changes. It is appropriate to take the rate of temperature dependency  $R(t)$  as temperature sensitivity of load. Temperature sensitivity represents the load increase when the temperature rises by 1 °C. Temperature sensitivity has a variable nature at seasonal, daily and hourly levels. As an example, the changes of temperature sensitivity at weekly and hourly levels are presented in Fig. 1 and Fig. 2. Temperature sensitivity, the unit of which is MW/°C is, in this case, negative – the rise of the temperature causes a fall of load and vice versa. The effect of the temperature is highest in winter. The peak in daily temperature sensitivity at 23.00 is apparently caused by switching on heating devices, due to the change in tariffs at that time.

Actually, the normalized component of temperature dependency  $\gamma(t)$  represents temperature deviation, unit °C. In the first approximation, temperature deviation may be found in relation to the normal temperature  $\Delta T(t) = T(t) - E[T(t)]$ , and thus  $\gamma(t) = \Delta T(t)$ . In practice, more precise presentation of the component  $\gamma(t)$  is necessary. It should enable more detailed description of the temperature dependency of the load, first of all considering the delay of the temperature influence (inertia). To present the normalized component of temperature dependency, it is appropriate to apply the time series *ARIMA* model (*Integrated Autoregressive – Moving Average Model*), which is also called *Box-Jenkins* model [5], as follows:

$$\gamma_t = \frac{\Psi_T(B)}{\Phi_T(B)} \Delta T_t.$$

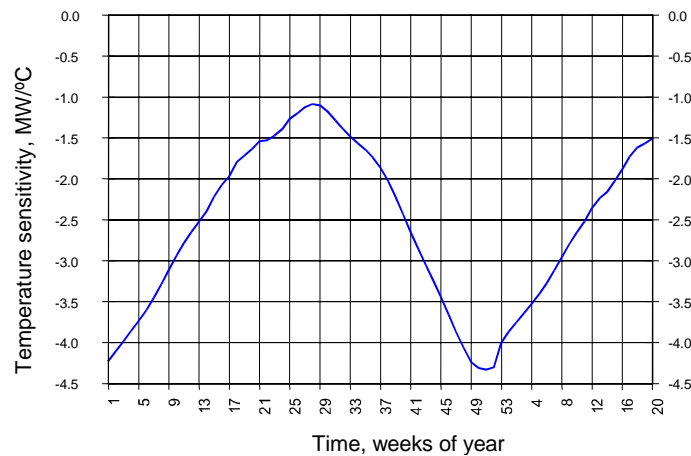


Fig. 1. Temperature sensitivity of the load, weekly values.

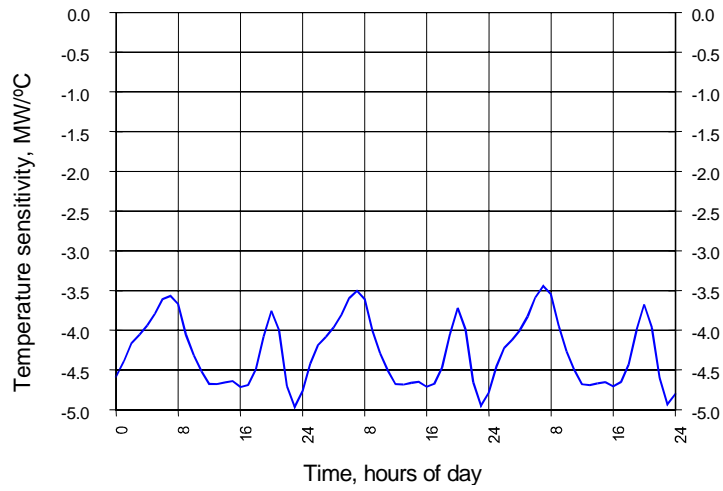


Fig. 2. Temperature sensitivity of the load, hourly values.

Here the operators  $\Phi_T(B)$  and  $\Psi_T(B)$  are the polynomials of the shift operator  $B$  ( $Bx_t = x_{t-1}$ ). If operators  $\Phi_T(B)$  and  $\Psi_T(B)$  are presented in the form of

$$\Phi_T(B) = 1 - \varphi B, \quad \Psi_T(B) = \psi B^m,$$

where  $\varphi$ ,  $\psi$  and  $m$  are parameters, then the component  $\gamma(t)$  corresponds to transfer function, shown in Fig. 3 with parameters

$$H_0 = m, \quad H = \frac{\varphi}{1 - \varphi}, \quad \gamma_\infty = \frac{\psi}{1 - \varphi}.$$

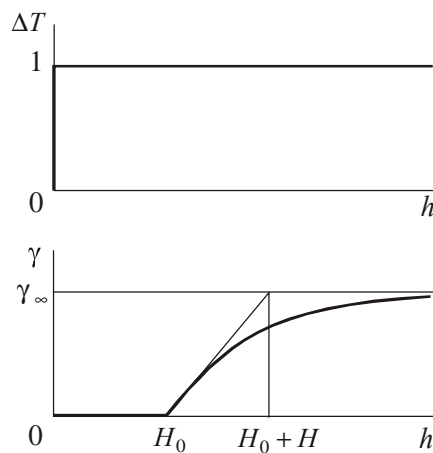


Fig. 3. Transfer function of temperature dependency.

According to this transfer function, the influence of temperature change on the load starts after  $H_0$  hours, where  $H$  is time constant of temperature dependency. For example, if  $H_0 = 5$  and  $H = 10$ , temperature change will affect the load fully after about  $H_0 + 2H = 25$  hours (i.e. on the next day).

In addition to level and inertia, one must also consider changes of temperature dependency in time and non-linearity. Time responds are taken into account by considering that model parameters change in time. It is essential to consider seasonal variability, because the influence of temperature in summer differs significantly from that in winter. If necessary, changes of weekly and daily temperature dependencies may also be taken into account. A non-linearity phenomenon occurs when, in the case of certain temperature values, the characteristics of temperature dependency change. Such phenomena are in conjunction with temperature deviation from a certain value instead of mathematical expectation of temperature. In the model, non-linearity is considered by adding the so-called marginal components, which will be activated if temperature falls below or reaches above the threshold values  $T_1$  or  $T_2$ . According to that, the component of temperature dependency  $\gamma(t)$  consists of three parts

$$\gamma_t = \gamma_{0t} + \gamma_{1t} + \gamma_{2t},$$

where

$$\begin{aligned} \Phi_{T_0}(B)\gamma_{0t} &= \Psi_{T_0}(B)(T_t - T_{0t}) \\ \Phi_{T_1}(B)\gamma_{1t} &= \begin{cases} \Psi_{T_1}(B)(T_t - T_1), & \text{if } T_t < T_1 \\ 0, & \text{if } T_t \geq T_1 \end{cases} \\ \Phi_{T_2}(B)\gamma_{2t} &= \begin{cases} \Psi_{T_2}(B)(T_t - T_2), & \text{if } T_t > T_2 \\ 0, & \text{if } T_t \leq T_2 \end{cases} \end{aligned}$$

Here  $T_{0t} = E[T_t]$  is mathematical expectation of the temperature.

If the delay in the case of marginal components is of the same level as the main components, the temperature dependency model is

$$\gamma_t = \varphi B \gamma_t + \psi_0 B^m (T_t - T_{0t}) + \psi_{T_1} B^m (T_t - T_1) \Big|_{T_t < T_1} + \psi_{T_2} B^m (T_t - T_2) \Big|_{T_t > T_2}.$$

Figures 4 and 5 are the examples of temperature dependency of the load with weekly and daily values [6]. For comparison, temperature and its mathematical expectation is also presented (scaled on the right axis). Here the load long-term forecast  $E(t) + R(t)\gamma(t)$  is achieved by adding temperature dependency to mathematical expectation. As temperature dependency is calculated basing on real temperature data, the expression "forecast" is somewhat conditional.

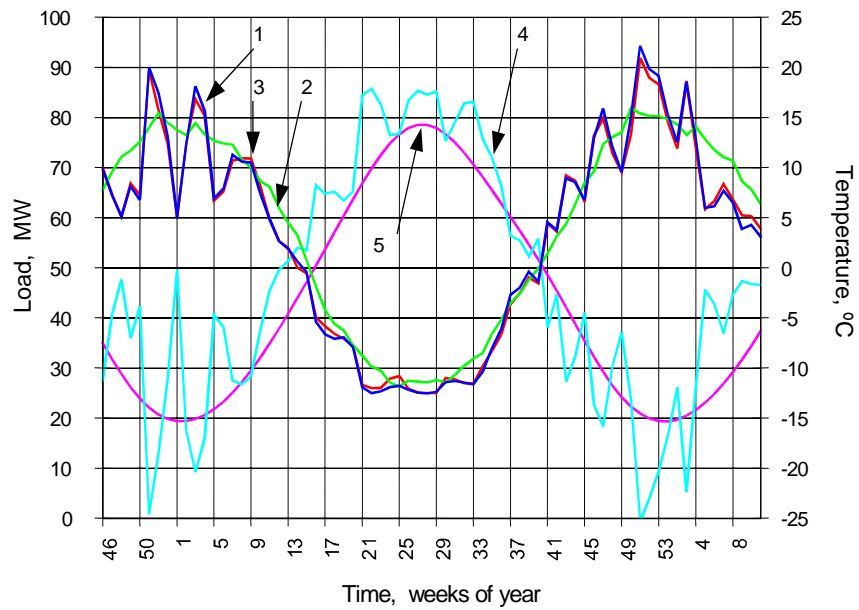


Fig. 4. Load real value (1), mathematical expectation (2), long-term forecast (3), temperature value (4), and normal temperature (5) (weekly values).

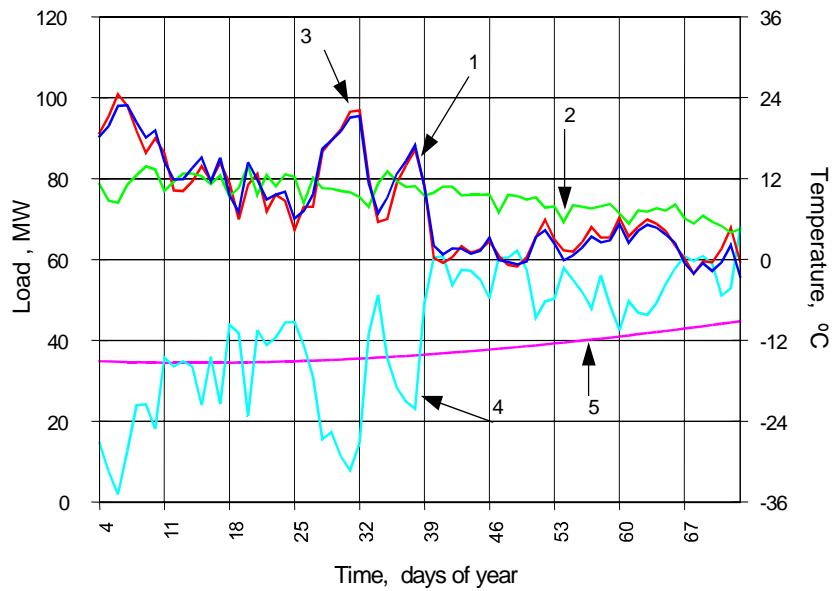


Fig. 5. Load real value (1), mathematical expectation (2), long-term forecast (3), temperature value (4), and normal temperature (5) (daily values).

Depending on the needed accuracy of modelling, other details of load temperature dependency may be considered. For example, load may not decrease in an ordinary way after a long, cold winter period (a week or more) when temperature returns to the normal level. A problem is how to present load temperature dependency of accumulative electrical heating. In that case temperature dependency is related not so much to the power of the heating system as to the duration of the turn-on time.

### Identification and estimation of the model

The components of the mathematical model are estimated using the hourly load data, which have been fixed at a certain data density, once or more times an hour. Yearly hourly data and temperature data of the load are needed to implement the estimation process. There should be considerably more data available for several years (5–10 years), in order to obtain reliable results. However, in a longer period the load characteristics may change, which prevents achieving more accurate results. Bad differentiation between mathematical expectation and temperature dependency of the load is also a problem in the estimation process. Sometimes it is difficult to determine whether load deviations mean the change of load character (mathematical expectation) or it occurs due to the influence of temperature.

Temperature sensitivity  $R(t)$  is estimated together with mathematical expectation  $E(t)$ . Mathematical expectation of the load principally corresponds to the normal temperature. Therefore, the estimation of the mathematical expectation is based on the data normalized in relation to the temperature

$$P'(t) = P(t) - R(t)\gamma(t).$$

In the estimation process of temperature sensitivity, load deviation in relation to mathematical expectation is used

$$\Delta P(t) = P(t) - E(t).$$

As the temperature dependency model is initially missing, an iteration process must be arranged where mathematical expectation and temperature sensitivity are adjusted after the evaluation of temperature dependency. Practically, 2–4 iterations are needed. The result is still not always reliable. Bad differentiation between mathematical expectation and temperature dependency arises. For example, when the observable time period is two years and both winters are soft, it is statistically difficult to determine the load level in a normal or a cold winter.

The model of the component  $\gamma(t)$  consists of seven parameters  $m, \varphi, \psi_0, \psi_1, \psi_2, T_1$  and  $T_2$ . The estimation is aggravated because of parameters changing with time. The nature of the marginal components must also be cleared. Bad differentiation between different components of the



model is also a problem. Thus, the parameters representing the delay of the temperature influence  $m, \varphi$  also affect the level of the temperature dependency. It is difficult to determine the relation of the main components of the model and marginal components basing on threshold temperatures  $T_1$  and  $T_2$ .

The parameters of the component  $\gamma(t)$  are estimated by characteristic time interval to consider temporal changes. It is necessary to divide the year at least into four periods (spring, summer, autumn and winter). For more precise modelling, it is possible to observe separately three periods of the week (workday, Saturday and Sunday) and four periods of the day (morning, hours in the middle of the day, the first hours of the night, the rest of the night).

The model parameters of temperature dependency are estimated in two phases. Firstly, the nature of the load temperature dependency is investigated, and the structure of the model (characteristic time periods, threshold temperatures of the marginal components etc.) and compatible loads are determined. In the second phase, applying the formal methods, the model parameters are estimated conclusively for every load. To enhance the reliability of the model, the component  $\gamma(t)$  may be estimated by load classes (temperature sensitivity  $R(t)$  is always adequate to the load). Similar response to temperature deviations is one of the bases for load classification.

The indication curves, functions of the residual of the normalized deviation of load dependency on temperature  $\Delta\theta(t) = f(\Gamma)$ , can be used as one instrument to research the temperature dependency. The residual is expressed as

$$\Delta\theta(t) = \frac{P(t) - E(t)}{R(t)} - \gamma(t),$$

where temperature deviation  $\gamma(t)$  is calculated on given model parameters. The residual value does not depend on temperature (the indication curve is practically horizontal) if the model parameters are appropriate to given conditions.

Figures 6 and 7 give examples of indication curves in the winter period for loads, which belong to the same class (summarized load of the regional electrical network). At first temperature dependency is not considered. The situation in the second case corresponds to the temperature dependency model with parameters  $m = 4$ ,  $\varphi = 0,89$  ( $H_0 = 4$  and  $H = 10$  hours),  $\psi_0 = 0,128$ ,  $\psi_{T_1} = -0,122$ ,  $\psi_{T_2} = 0,017$ ,  $T_1 = -25$  and  $T_2 = 0$ . The marginal components of the model indicate that when the temperature goes below  $-25$  °C, the load increase will decelerate and, respectively, it will accelerate when temperature rises above  $0$  °C.

Basing on the normal temperature, which in weather stations is mostly found as 30-year mean values for every hour, it is necessary to model the mathematical expectation of the temperature. The estimation of the mathematical expectation means the approximation of the normal temperature in a way appropriate to the model.

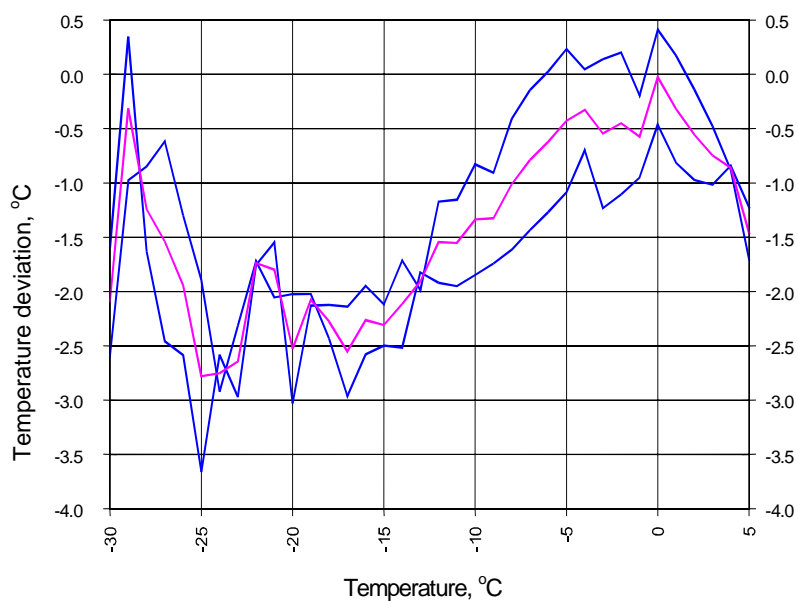


Fig. 6. Preliminary indication curves of the load.

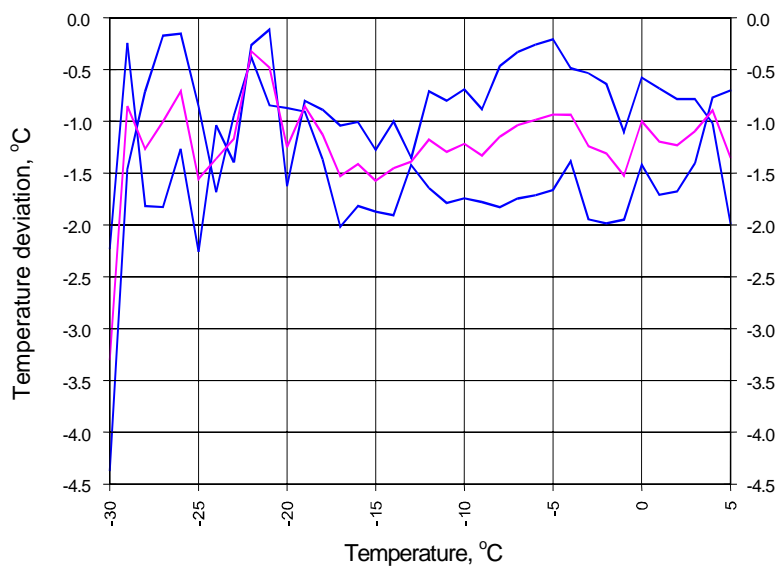


Fig. 7. Indication curves when considering temperature dependency.

## Conclusions

In the load monitoring process of the electrical network, it is essential to consider the influence of outdoor temperature on the load. Load deviations

caused by temperature may be considerable in conditions where electrical heating or air conditioning devices are used. The mathematical model considers the extent of load deviations caused by temperature, and also the delay of the temperature influence, non-linearity and temporal changes.

The structure of the mathematical model is the same for all loads. During estimation of the model parameters, problems occur which are associated with slow changes in weather conditions, low volume of initial data, and bad determinancy of the model components. In order to obtain reliable results, the models are initially identified – typical cases of temperature dependency of the load are determined and some of the model parameters are fixed. The final estimation of the model takes place using the methods of computational mathematics depending on the level of the initial data and required accuracy of the model.

First and foremost, the temperature dependency model may be used for calculating the temperature influence in the load analysis and short-term forecast processes, when the real or meteorologically forecasted values of temperature are known. In long-term forecasting of the load the temperature values may be simulated. The temperatures measured earlier in a situation characteristic of the present case (cold winter etc.) may be used for the simulation purposes. The deviation from the normal temperature may also be given, and the corresponding values of load deviation may be found.

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