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OPTIMAL PLANNING OF GENERATING UNITS IN POWER SYSTEM CONSIDERING UNCERTAINTY OF INFORMATION

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The task of optimal planning of generating units (OPGU) for optimal operation and expansive planning of all-thermal power system is considered in this paper. The OPGU problem is dealt with as a two-stage problem. During the first stage the optimal dispatching of units when the unit commitment is given takes place. During the second stage the optimal combination of units considering optimal dispatching of units on the first stage will be determined. Taking into consideration the incompleteness of information, besides the deterministic models of OPGU the min-max models will be discussed. Also some procedures of solving the tasks of OPGU and illustrative examples are presented.

Introduction

The problem of optimal planning of generating units (OPGU) arises in optimal operation of power system as well as in planning of new units for expanding power system. This is one of the most important optimization problems in power systems. The OPGU problem consists of two subproblems: 1) optimization of dispatching, 2) optimization of unit commitment.

In optimal operation of a power system time horizons of problems are minutes, hours, days, weeks, months and year. The objective of optimization is to minimize the instant or expected cost of fuel or operation [1–6].

For planning new generating units the time horizon must be within the time frame of years (commonly 5–30 years) and the purpose of optimization is commonly minimizing the expected investment and operational costs [6].

Today the differences between the mathematical models of OPGU used in optimal operation and in expanding planning are great. If the models of optimal operation are non-linear and often also stochastic, mainly deterministic and linear models are used in planning of new units. Actually the problems of OPGU are non-linear, and the initial information for them is incomplete.

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In this paper the problem of OPGU in all-thermal systems will be dealt with as a two-stage non-linear optimization problem. The deterministic and min-max models of OPGU will be presented below. Also some procedures for solving deterministic and min-max tasks of OPGU are described and illustrative examples are added.

Deterministic models of OPGU

Active power demand of a power system is continually changing. The changes in power demand may be described by load demand curves or load demand duration curves [4]. In power system operation in the case of problems with time horizon up to weeks, load demand curves are commonly used. However, in the long-term planning problems, the load demand is described by load demand duration curves.

The nomenclature used in this paper is as follows:

- F – total operation cost or investment and operational costs on the power system during the period T
- $P_{Gi}(t)$ – net active power output for unit i in t^{th} time interval
- $P_G(t) = \langle P_{G1}(t), i = 1,...,N; t = 1,...,T \rangle$ – vector of net active power outputs of units in the period T
- $P_D(t)$ – net system active power demand in the time period T
- $P_L(t)$ – total losses of active power in the t-th time interval
- $v_i(t)$ – commitment state of the unit i in t-th time interval:
  $v_i(t) = 1$ – generating unit i is operating
  $v_i(t) = 0$ – generating unit i is off
- $V(t) = \langle v_i(t), i = 1,...N, t = 1,...,T \rangle$ – vector of commitment states in the period T
- $S_i(t)$ – start-up and banking costs of the unit i in the t time interval
- $k(t)$ – number of hours within the t-th time interval
- $P^0_{Gi}(V(t), t)$ – value of net output of the unit i at hour t optimized by the first submodel
- $P^+_R(t), P^-_R(t)$ – necessary spinning reserves planned to the "+" and "–" directions in the t-th time interval.

Assuming that all initial information for OPGU is given in the deterministic form, the OPGU problem may be written as a deterministic two-stage problem

$$\min_V \min_{P_G} F(V, P_G, P_D) \tag{1}$$

subject to the constraints of OPGU.

Subproblem 1 – Optimal Dispatching of Units

During the first stage the problem of optimal dispatching of units will be solved:

$$\min_{P_G} F(V(t), P_G(t)) = \sum_{t=1}^{T} k(t) \cdot \sum_{i=1}^{N} \left[v_i(t) F_i(P_{Gi}(t)) \right] \tag{2}$$

subject to:

1) system power balance equations

$$P_D(t) + P_L(t) - \sum_{i=1}^{N} v_i(t) P_{Gi}(t) = 0 \quad \text{for } t = 1,...,T \tag{3}$$

2) unit generation limits

$$P_{Gi}^{\min} \leq P_{Gi}(t) \leq P_{Gi}^{\max} \qquad \text{for } t = 1,...,T \text{ and } i = 1,\dots,N \tag{4}$$

The problem of optimal dispatching will be solved for a given unit commitment $V(t)$ and for a given load demand curve $P_D(t)$.

The Lagrangian function for problem (2)–(4) may be expressed as

$$\Phi(P_G(t), V(t), P_D(t), \mu(t)) =$$
$$\sum_{t=1}^{T} k(t) \cdot \sum_{i=1}^{N} \left[v_i(t) F_i(P_{Gi}(t)) \right] - \sum_{t=1}^{T} \mu(t) \left[P_D(t) + P_L(t) - \sum_{i=1}^{N} v_i(t) P_{Gi}(t) \right] \tag{5}$$

Now, the problem of optimal dispatching can be written as

$$\min_{P_G(t)} \max_{\mu(t)} \Phi(P_G(t), V(t), P_D(t), \mu(t)) \tag{6}$$

subject to constraints (4).

Subproblem 2 – Optimal Unit Commitment

On the second stage the problem of optimal unit commitment will be solved:

$$\min_{V(t)} F(V(t)) = \sum_{t=1}^{T} k(t) \cdot \sum_{i=1}^{N} \left[v_i(t) F_i(P_{Gi}^0(V,t)) + S_i(t) \right] \tag{7}$$

subject to:

1) spinning reserve requirements

$$P_D(t) + P_L(t) + P_R^+(t) - \sum_{i=1}^{N} v_i(t) P_{Gi}^{\max} \leq 0 \quad \text{for } t = 1,...,T \tag{8}$$

$$\sum_{i=1}^{N} v_i(t) P_{Gi}^{\min} - P_D(t) - P_L(t) + P_R^-(t) \leq 0 \quad \text{for } t = 1,...,T \tag{9}$$

2) unit generation limits

$$P_{Gi}^{\min} \leq P_{Gi}^0(V,t) \leq P_{Gi}^{\max} \qquad \text{for } t = 1,...,T,\ i = 1,\dots,N \text{ and if } v_i(t) = 1 \tag{10}$$

The Lagrangian function of the second subproblem has the following form:

$$H(V(t), P_D(t), P_R^+(t), P_R^-(t), \lambda_1(t), \lambda_2(t)) = \sum_{t=1}^{T} k(t) \cdot \sum_{i=1}^{N} [v_i(t) F_i(P_{Gi}(V(t), t) + S_i(t)] +$$
$$\sum_{t=1}^{T} \left[\lambda_1(t)(P_D(t) + P_L(t) + P_R^+ - \sum_{i=1}^{N} v_i(t) \cdot P_{Gi}^{\max}) \right] +$$
$$\sum_{t=1}^{T} \left[\lambda_2(t)(\sum_{i=1}^{N} v_i(t) P_{Gi}^{\min} - P_D(t) - P_L(t) + P_R^-(t)) \right] \quad (11)$$

Now, using the Kuhn-Tucker conditions [6] and the Lagrangian function (11), the problem of optimal unit commitment can be presented in the following form:

$$\min_{V(t)} \max_{\lambda_1(t)} \max_{\lambda_2(t)} H(V(t), P_D(t), P_R^+(t), P_R^-(t), \lambda_1(t), \lambda_2(t)) \quad (12)$$

subject to the constraints (10) and

$$\lambda_1(t) \cdot (\sum_{i=1}^{N} v_i(t) P_{Gi}^{\max} - P_D(t) - P_L(t) - P_R^+(t)) = 0 \quad (13)$$

$$\lambda_2(t) \cdot (P_D(t) + P_L(t) - P_R^-(t) - \sum_{i=1}^{N} v_i(t) P_{Gi}^{\min}) = 0 \quad (14)$$

$$\lambda_1(t) \geq 0 \quad (15)$$

$$\lambda_2(t) \geq 0 \quad (16)$$

The subproblem 2 will be solved assuming that the subproblem 1 will be solved for given values of $V(t)$ and $P_D(t)$.

The cost function of unit consists of fixed and variable costs:

$$F_i(P_{Gi}) = F_i^{Fix} + F_i^{Var}(P_{Gi} - P_{Gi}^{\min}) \quad (17)$$

where F_i^{Fix} – fixed costs of i^{th} unit;

$F_i^{Var} = F_i^{Var}(P_{Gi} - P_{Gi}^{\min})$ – variable costs of i^{th} unit.

The cost functions of units must be as follows:

1. For minimizing the costs of fuel in power system operation and planning, in place of functions $F_i(P_{Gi})$ one must use the fuel input-output characteristics of units, where F_i^{Fix} is the cost of fuel if $P_{Gi} = P_{Gi}^{\min}$.
2. For minimizing the operational cost in power system operation and planning, in place of functions $F_i(P_{Gi})$ one must use the characteristics of operational cost of units, where F_i^{Fix} is the fixed operational cost and $F_i^{Var}(P_{Gi} - P_{Gi}^{\min})$ is the variable cost of fuel.

3. For planning of new units, in place of functions $F_i(P_{Gi})$ one must use the characteristics of operational cost of units, where F_i^{Fix} is the total fixed operational and investment costs and $F_i^{Var}(P_{Gi} - P_{Gi}^{\min})$ is the variable cost of fuel.

Procedure of Solving OPGU problems

A computer program for optimal solving OPGU is developed. The program is composed in Compaq Visual Fortran. Database is in EXCEL.

Subproblem 1 will be solved by μ-iteration method when the variables $V(t)$ and $P_D(t)$ are given.

The initial data, needed for solving the subproblem 1, are: $P_D(t)$, $P_{Gi}^{\min}$, $P_{Gi}^{\max}$, $F_i(P_{Gi})$ for i =1, …, N, t=1, …, T and formulas for losses.

For solving the subproblem 2 the method of forward dynamic programming is used. The initial data needed for optimization of unit commitment are: $P_D(t)$, $k(t)$, $P_R^+(t), P_R^-(t)$, $P_{Gi}^{\min}$, $P_{Gi}^{\max}$, formulas for losses, the set of units and it's characteristics for t =1, …, T.

In power system control and short-term planning it is necessary to take into account also start-up and banking costs and several technical constraints of unit commitment.

Unfortunately, initial information for deterministic models of OPGU is comparatively inexact. Therefore the problems of OPGU had to be solved under incomplete information. For this, the deterministic models of optimization must be replaced by the probabilistic models or by models that can optimize generating units under uncertainty and fuzzy conditions.

Min-Max Models of OPGU

In this paper uncertainty of information means that only intervals of variables and intervals of values of functions are given but not their concrete values. In the given intervals the values of variables and functions are uncertainties. There are several possibilities to optimize systems under uncertainty conditions. Different approaches will arise depending on what criterion of optimality will be used. We have studied availabilities of Laplace criterion, Hurwicz criterion, min-max cost and min-max regret criterion [7–9]. The best criterion for OPGU problem under uncertainty conditions is min-max regret. Min-max regret criterion was advocated by Savage in 1954 [10].

We will name the min-max regret criterion the criterion of min-max risk or losses caused by uncertainty of information, because this criterion guarantees that maximum of losses from the uncertainty of information is as small as possible.

Define the risk function caused by uncertainty of information:

$$R(\overline{Y},Z)=F(\overline{Y},Z)-\min_{Y}F(Y,Z) \tag{18}$$

where $\min_{Y}F(Y,Z)$ – minimum total costs if optimization will taken place under complete information;
Y – vector of controllable variables;
Z – vector of non-controllable variables.

The value of function (18) shows the losses that arise if vector $\overline{Y}$ is not optimal solution of deterministic task.

For optimization under uncertainty we must solve the following problem of the min-max risk

$$\min_{Y}\max_{Z}R(\overline{Y},Z) \tag{19}$$

subject to the corresponding constraints.

The main uncertainty factors in the OPGU are the system load demand $P_D(t)$ and the input-output characteristics of units.

Assume that we know the intervals of load demand and input-output characteristics of the unit:

$$P_D^-(t)\le P_D(t)\le P_D^+(t) \tag{20}$$

and

$$F_i^-(P_{Gi})\le F_i(P_{Gi})\le F_i^+(P_{Gi})\,,\quad i=1,\ldots,N \tag{21}$$

or

$$\beta_i^-(P_{Gi})\le \beta_i(P_{Gi})\le \beta_i^+(P_{Gi}) \tag{22}$$

where $\beta_i=\dfrac{\partial F_i}{\partial P_{Gi}}$ – is the characteristic of incremental cost rate of *i*-th unit.

Let $P_D^-(t),P_D^+(t)$ and $F_i^-(P_{Gi}),F_i^+(P_{Gi})$ or $\beta_i^-(P_{Gi}),\beta_i^+(P_{Gi})$ are given.

The conditions of optimality for the tasks of min-max risk (19) may be derived from the main theorem of game theory with convex functions [10]. For solution of min-max problem OPGU it is recommended to use the two-stage approach as well [7]. During the first stage the deterministic equivalent of min-max problem will be found and during the second stage the common deterministic optimization problem with modified functions will be solved.

Examples

1. Optimization of Power Units on the Basis of Load Duration Curve

The load curve and load duration curve for one year are shown in Figures 1 and 2.

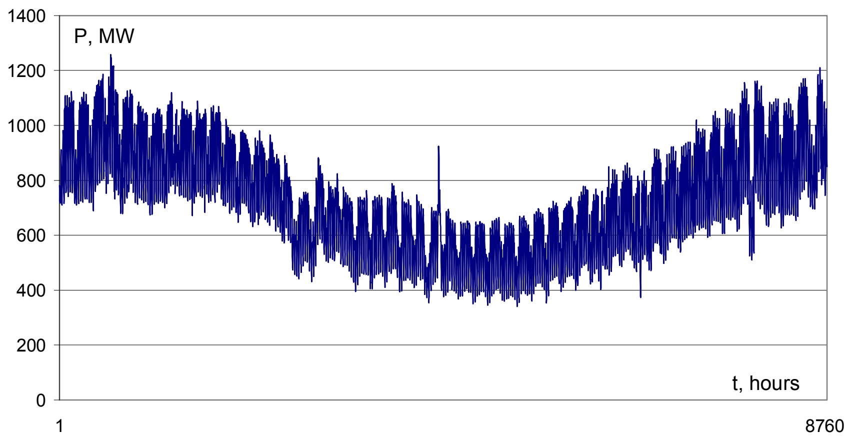

Fig. 1. Annual electric load curve

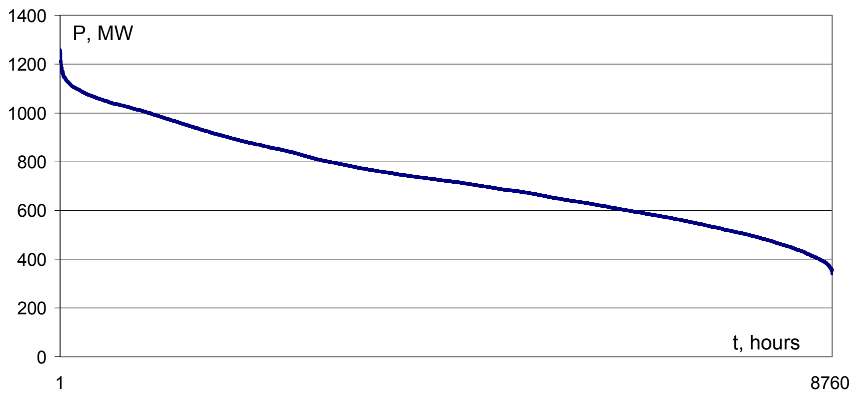

Fig. 2. Annual load duration curve

The power system load is often divided into three categories:
1) Base load (duration time 8,760 hours in a year)
2) Intermediate load (duration time from 2,000 to 8,760 hours in a year)
3) Peak load (duration time up to 2,000 hours in a year).

In the Estonian power system, the base load forms about 35%, intermediate load about 40% and peak load about 25% of the maximum load. The power system must have sufficient active and reactive power-generating capacity to cover the load changes since the electricity cannot be conveniently stored in sufficient quantities. Therefore the power system must have the following types of generating units:

1) Base-load generating units
2) Intermediate load-generating units
3) Peak-load generating units
4) Frequency and power control units.

Let there be four types of power units in a power system:

1) oil-shale units (OU) – 2 items
2) coal units (CU) – 2 items
3) gas units (GU) – 2 items
4) gas turbine (GT) – 1 item.

Input-output characteristics of these types of units are shown in Fig. 3; and optimal covering of load demand duration curve is presented in Fig. 4.

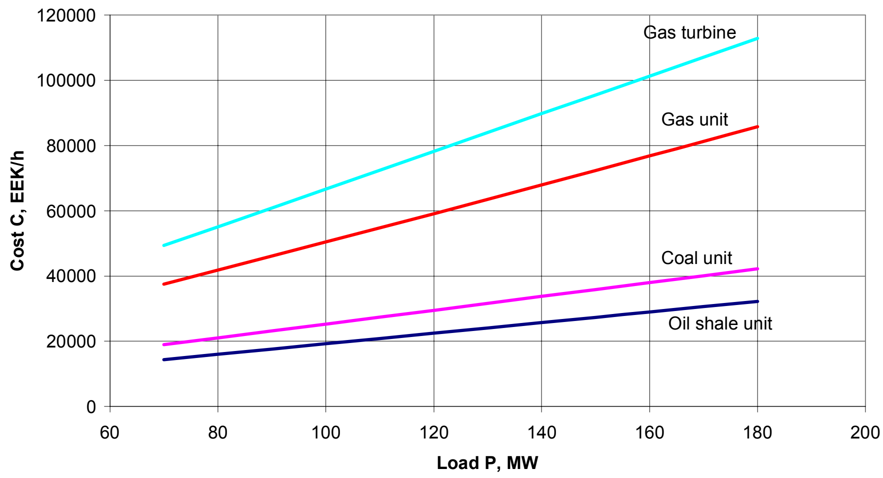

Fig. 3. Fuel cost characteristics of different types of power units

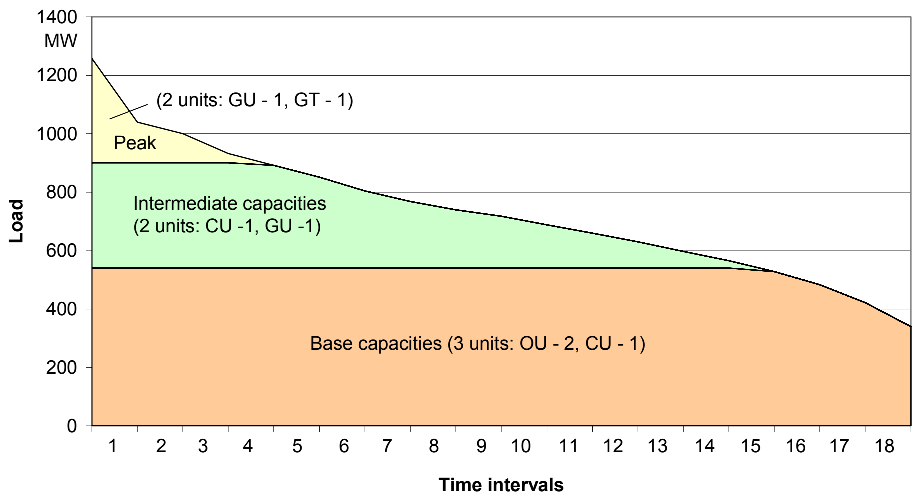

Fig. 4. Optimal covering of load demand duration curve

2. Optimization under Uncertainty of Load Demand

For transforming the min-max tasks to the deterministic tasks we must find the load demand value $\overline{P}_D(t)$, when

$$\min_{\overline{Y}} R(\overline{Y}, Z^+) = \min_{\overline{Y}} R(\overline{Y}, Z^-) \tag{23}$$

The economical risks caused by uncertainty of load demand are illustrated in Fig. 5. The uncertainty interval of load is 245–295 MW.

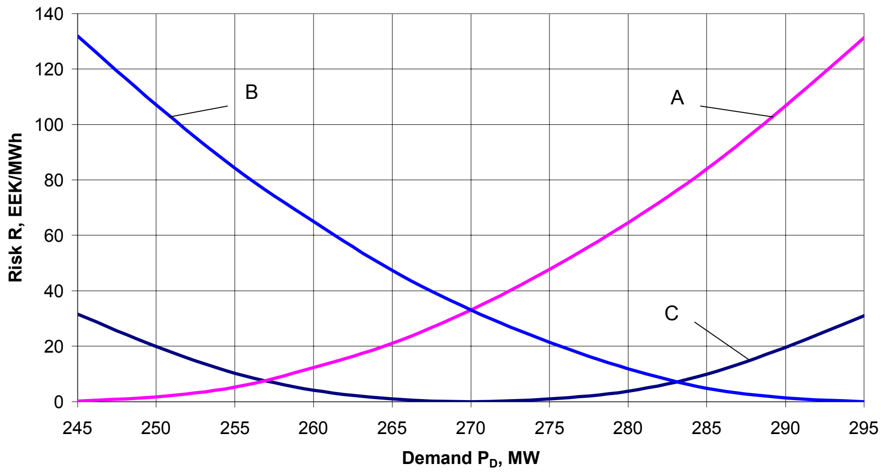

Fig. 5. Economical risks, caused by uncertainty of load demand:
A – if $\overline{P}_D = 245$ MW, B – if $\overline{P}_D = 295$ MW, C – if $\overline{P}_D = 270$ MW

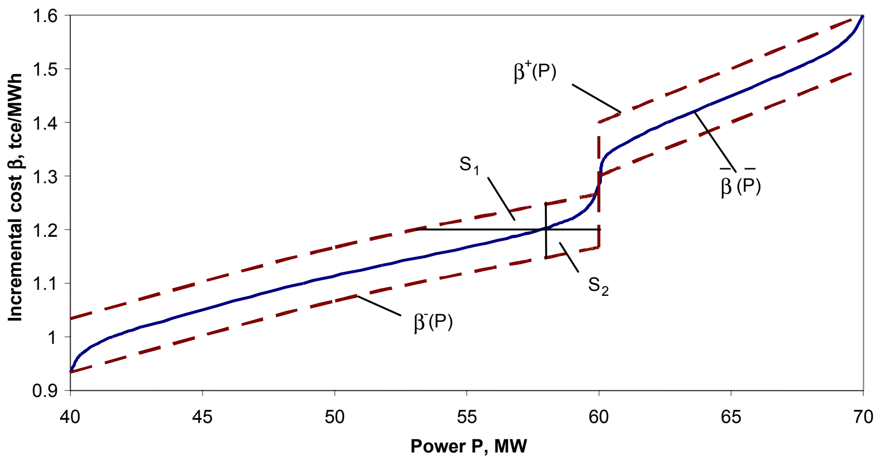

Fig. 6. Characteristics of incremental fuel cost (measured in tonnes of coal equivalent per MWh). – – – initial lower and upper characteristics, —— planned characteristic

If to plan the power demand to the center of the interval of uncertainty, the maximum risk will be about 4 times smaller than in the case when the power demand will be planned to the border points of the interval of uncertainty.

3. Optimization under Uncertainty of Input-Output Characteristics

To consider the uncertainty of unit's characteristics it is necessary to determine such planned characteristics for units that will correspond to conditions (22). An example of the planned characteristic is shown in Fig. 6.

The using of planned characteristics in place of lower and upper initial characteristics will decrease the maximum of risk function approximately four times.

Conclusions

1. A united two-stage method of OPGU for optimal control and expanding planning of power systems is presented in the paper.
2. The method uses the non-linear models and enables to optimize:
   - The unit commitment for existing units and for choosing news ones
   - Generating units on the basis of load demand curves or load demand duration curves
   - Generating units on the basis of uncertain information about load demand and characteristics of units.
3. An experimental program has been composed, and a corresponding software system for power companies is being currently designed.
4. The new software system will be substantially more universal and effective than software systems known at present.

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