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OPTIMAL OPERATION OF POWER PLANTS IN COGENERATION SYSTEMS

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This paper presents the principles of optimal dispatch of heat and electrical power: both the district heating and electrical power systems are connected with cogeneration power plants. The cogeneration system with condensing power plants, boiler plants and cogeneration plants with extraction and back-pressure turbines are considered. The conditions of optimality using Lagrangian function are presented. In addition to the combined optimization of heat and power the three subtasks are tackled. These subtasks consider the optimal load dispatch in different market conditions. A computer program has been developed.

Introduction

In general, a cogeneration power system consists of one power (electric power) system and several heating systems. The problems of optimal dispatch of the heat and power in such systems differ somewhat from conventional economic dispatch problem of electric power system.

According to the Directive 2004/8/EC of the European Parliament and of the Council on the promotion of cogeneration based on useful heat demand in the internal energy market, a Community priority is given over the potential benefits of cogeneration with regard to saving primary energy, avoiding network losses and reducing emissions. The cogeneration can also contribute positively to the security of energy supply.

Several works have been written on this topic [1–5]. However, there are still many aspects of the combined optimal dispatch of heat and power generation in a cogeneration system, which are unsolved or not sufficiently explained.

It is very important to work out the models and the method for short- and long-term planning of the operation of cogeneration stations in a power system.

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A deterministic approach to the solution of the economic dispatch problem in the cogeneration power system is considered. The network losses, transmission costs and emissions are not considered here.

A cogeneration system observed here consists of conventional condensing units (P units), heat production units (boilers) (Q units) and cogeneration units (PQ units) (Fig. 1). The cogeneration units may have the automatic-extraction or back-pressure turbines.

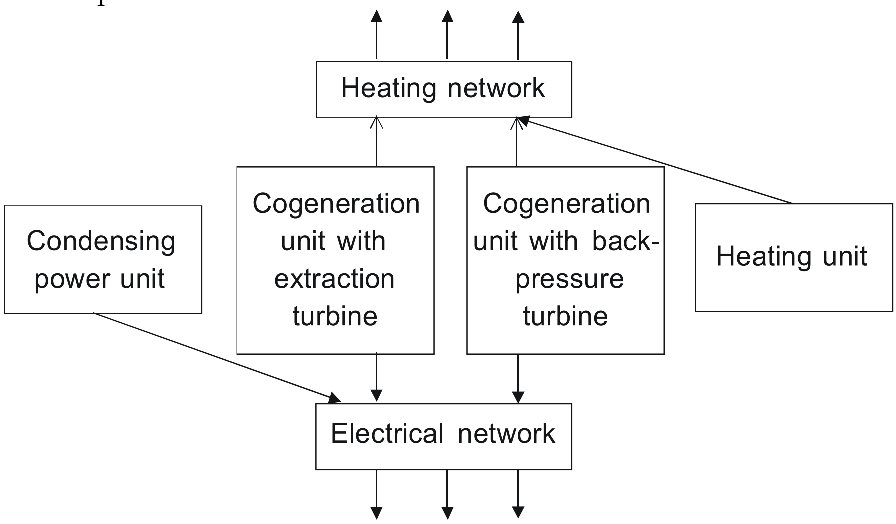

Fig. 1. Cogeneration system

Characteristics of Power Plants

Condensing Power Plant: Steam Boiler & Condensing Turbine & Generator (P Unit)

The input-output characteristics of a condensing power plant can be presented as a composite function:

$$C_P(P_P) = C_B(Q_T(P_P)) \tag{1}$$

where C_P – fuel cost of the condensing power plant;

P_P – power output of the condensing power plant;

$C_B(*)$ – fuel cost function of the boilers;

Q_T – steam input to the turbines;

$Q_T(P_P)$ – input-output characteristics of the condensing turbine.

The condensing power plants have usually continuous, piecewise smooth and strictly convex cost functions.

Heat Production Unit: Hot-Water Boiler (*Q* Unit)

Hot-water boilers have a smooth and strictly convex fuel cost function:

$$C_Q(Q_Q) = C_B(Q_Q) \tag{2}$$

where C_B – fuel cost of the boiler unit;

Q_Q – heat output of the Q unit.

Cogeneration Power Plant:
Boilers & Extraction Turbines & Generators (*PQ* Unit)

The input-output characteristics of cogeneration power plants (CPP) with the extraction turbines can be presented as follows:

$$C_{PQE}(Q_{QE}, P_P) = C_B(Q_{PQE}(Q_{QE}, P_P)) \tag{3}$$

$$Q_{PQE}(Q_{QE}, P_P) = Q_{PQET}(Q_{QE}) + Q_{PT}(P_P) \tag{4}$$

$$P_{PQE} = P_{PQE}(Q_{QE}) \tag{5}$$

$$P_{PQ} = P_{PQE} + P_P \tag{6}$$

where C_{PQE} – fuel cost of the CPP with the extraction turbine;

$C_B(*)$ – fuel cost function of the boiler;

$Q_{PQE}(*)$ – input-output diagram of the extraction turbine;

P_P – condensing part of the power output of the extraction turbine;

$P_{PQE}(*)$ – cogeneration part of the power output of the extraction turbine;

Q_{QE} – heat load (output) of the extraction turbine;

P_{PQ} – total power generation output of the CPP with the extraction turbine.

Cogeneration Power Plant:
Boiler & Back-Pressure Turbine & Generator (PQBP Unit)

The input-output characteristics of the CPP with the back-pressure turbines can be presented as follows:

$$C_{PQBP}(Q_{PQBP}) = C_B(Q_{BPT}(Q_{PQBP})) \tag{7}$$

$$P_{PQBP} = P_{PQBP}(Q_{PQBP}) \tag{8}$$

where C_{PQBP} – fuel cost of the CPP with back-pressure turbine;

$C_B(*)$ – fuel cost function of the boiler;

$Q_{BPT}(*)$ – input-output diagram of the back-pressure turbine;

Q_{PQBP} – heat load or output of the back-pressure turbine;

The cogeneration power output equals the total power generation of the back-pressure turbine.

Optimal Dispatch Problem

Basic Formulation

The optimal dispatch problem is determination of the loads of power units so that the cogeneration system production cost is minimized and all constraints are met.

The goal is to minimize the following objective function:

$$\min_{P_P, Q_{PQE}, Q_{PQBP}, Q_Q} C = \sum_{i=1}^{n1} C_{P,i} + \sum_{j=1}^{n2} C_{Q,j} + \sum_{k=1}^{n3} C_{PQE,k} + \sum_{l=1}^{n4} C_{PQBP,l} \tag{9}$$

subject to

1) system-wide constraints – the power and heat balance equations:

$$G_P = P_D - \sum_{i=1}^{n1} P_{P,i} - \sum_{k=1}^{n3} (P_{P,k} + P_{PQE,k}(Q_{PQE,k}) - \sum_{l=1}^{n4} P_{PQBP,l}(Q_{PQBP,l}) = 0 \tag{10}$$

$$G_Q = Q_D - \sum_{j=1}^{n2} Q_{Q,j} - \sum_{k=1}^{n3} Q_{PQE,k} - \sum_{l=1}^{n4} Q_{PQBP,l} = 0 \tag{11}$$

2) constraints of condensing plants ($i = 1,\ldots,n1$)

$$P_{P,i}^{\min} \le P_{P,i} \le P_{P,i}^{\max} \tag{12}$$

3) constraints of boiler plants ($j = 1,\ldots, n2$)

$$Q_{Q,j}^{\min} \le Q_{Q,j} \le Q_{Q,j}^{\max} \tag{13}$$

4) constraints of CPP with extraction turbines ($k = 1,\ldots, n3$)

$$Q_{PQE,k}^{\min} \le Q_{PQE,k} \le Q_{PQE,k}^{\max} \tag{14}$$

$$P_{P,k}^{\min} \le P_{P,k} \le P_{P,k}^{\max} \tag{15}$$

5) constraints of CPP with back-pressure turbines ($l = 1,\ldots, n4$)

$$Q_{PQBP,l}^{\min} \le Q_{PQBP,l} \le Q_{PQBP,l}^{\max} \tag{16}$$

Here P_D is the electric power demand; Q_D is the heat demand; i, j, k, l are the indices and $n1, n2, n3, n4$ are the numbers of the condensing power plants, boiler plants, CPP with extraction turbines and CPP with back-pressure turbines, respectively. "*Min*" and "*max*" mean the minimum and maximum values of the parameters.

Conditions of Optimality

The Lagrangian function of the problem is

$$\Phi = C + \lambda_P G_P + \lambda_Q G_Q \tag{17}$$

where λ_P, λ_Q are Lagrangian multipliers.

By the use of the Lagrangian function the problem of combined heat and power economic dispatch can be written as follows:

$$\min_{P_P, Q_Q, Q_{PQE}, Q_{PQBP}} \max_{\lambda_P, \lambda_Q} \Phi \tag{18}$$

If the point Y^0 is the optimum solution of the problem, the conditions for the optimum are:

1) for condensing units $(i = 1, \ldots, n1)$

$$\frac{\partial \Phi}{\partial P_{P,i}} = \frac{\partial C_{P,i}}{\partial P_{P,i}} - \lambda_P^0 \begin{cases} = 0 & if \quad P_{P,i}^{\min} < P_{P,i}^0 < P_{P,i}^{\max} \\ \geq 0 & if \quad P_{P,i}^0 = P_{P,i}^{\min} \\ \leq 0 & if \quad P_{P,i}^0 = P_{P,i}^{\max} \end{cases} \tag{19}$$

2) for boiler units $(j = 1, \ldots, n2)$

$$\frac{\partial \Phi}{\partial Q_{Q,j}} = \frac{\partial C_{Q,j}}{\partial Q_{Q,j}} - \lambda_Q^0 \begin{cases} = 0 & if \quad Q_{Q,j}^{\min} < Q_{Q,j}^0 < Q_{Q,j}^{\max} \\ \geq 0 & if \quad Q_{Q,j}^0 = Q_{Q,j}^{\min} \\ \leq 0 & if \quad Q_{Q,j}^0 = Q_{Q,j}^{\max} \end{cases} \tag{20}$$

3) for cogeneration units with extraction turbines $(k = 1, \ldots, n3)$

$$\frac{\partial \Phi}{\partial P_{P,k}} = \frac{\partial C_{PQE,k}}{\partial P_{P,k}} - \lambda_P^0 \begin{cases} = 0 & if \quad P_{P,k}^{min} < P_{P,k}^0 < P_{P,k}^{max} \\ \geq 0 & if \quad P_{P,k}^0 = P_{P,k}^{min} \\ \leq 0 & if \quad P_{P,k}^0 = P_{P,k}^{max} \end{cases} \tag{21}$$

$$\frac{\partial \Phi}{\partial Q_{PQE,k}} = \frac{\partial C_{PQE,k}}{\partial Q_{PQE,k}} - \lambda_P \frac{\partial P_{PQE,k}}{\partial Q_{PQE,k}} - \lambda_Q \begin{cases} = 0 & if \quad 0 < Q_{PQE,k}^0 < Q_{PQE,k}^{\max} \\ \geq 0 & if \quad Q_{PQE,k}^0 = 0 \\ \leq 0 & if \quad Q_{PQE,k}^0 = Q_{PQE,k}^{\max} \end{cases} \tag{22}$$

4) for cogeneration units with back-pressure turbines ($l = 1,\ldots, n4$)

$$\frac{\partial \Phi}{\partial Q_{PQBP,l}} = \frac{\partial C_{PQBP,l}}{\partial Q_{PQBP,l}} - \lambda_P \frac{\partial P_{PQBP,l}}{\partial Q_{PQBP,l}} - \lambda_Q \begin{cases} = 0 & if \quad Q_{PQBP,l}^{\min} < Q_{PQBP,l}^{0} < Q_{PQBP,l}^{\max} \\ \geq 0 & if \quad Q_{PQBP,l}^{0} = Q_{PQBP,l}^{\min} \\ \leq 0 & if \quad Q_{PQBP,l}^{0} = Q_{PQBP,l}^{\max} \end{cases} \tag{23}$$

5) for determination the Lagrangian multipliers

$$\frac{\partial \Phi}{\partial \lambda_P} = G_P = 0 \tag{24}$$

$$\frac{\partial \Phi}{\partial \lambda_Q} = G_Q = 0 \tag{25}$$

The superscript "0" of the symbols marks the optimal value of the parameter.

The optimal solution of the problem (9)–(16) must satisfy the conditions (19)–(25).

The Equations (19), (21) and (24) are the so-called coordination equations for the power system, and the Equations (20), (22), (23) and (25) are the coordination equations for the heating system. The derivations are the incremental rates of parameters and the marginal costs of the units.

Subtasks

Not every time do the cogeneration plants take part in the combined optimization on the cogeneration system level. Often the operation of cogeneration power plants is optimized only on the heating system (district heating system) or power system levels, or the cogeneration plants act as independent power plants on the heat and power markets.

Therefore the following three subtasks are of practical interest as well.

Subtask 1

If the cogeneration plants do not participate in the optimization on the power system level, the optimization of the heat and power loads is based on the following objective function:

$$\min_{Q_{PQE}} \left[\sum_{j=1}^{n2} C_{Q,j} + \sum_{k=1}^{n3} (C_{PQE,k} - h_P \cdot (P_{PQE,k}(Q_{PQE,k}) + P_{P,k}) + \sum_{l=1}^{n4} (C_{PQBP,l} - h_P \cdot P_{PQBP,l}(Q_{PQBP,l})) \right] \tag{26}$$

where h_P is the price of electricity.

Subtask 2

If the cogeneration power plants do not take part in the optimization on the heating system level, the objective function is as follows:

$$min \left[\sum_{k=1}^{n3} (C_{PQE,k} - h_Q \cdot Q_{PQE,k} + \sum_{l=1}^{n4} (C_{PQBP,l} - h_Q \cdot Q_{PQBP,l}) \right] \tag{27}$$

where h_Q is the price of heat.

This subtask enables to maximize electricity production of CPP in the cogeneration system.

Subtask 3

If the cogeneration power plants act as independent producers on the heat and power markets, they can optimize their operation according to the following objective functions:

- *CPP* with extraction turbines:
  to minimize the costs

$$min \sum_{k=1}^{n3} (C_{PQE,k} - h_Q \cdot Q_{PQE,k} - h_P \cdot (P_{P,k} + P_{PQE,k})) \tag{28}$$

or maximizing the profit

$$max \sum_{k=1}^{n3} (h_Q \cdot Q_{PQE,k} + h_P \cdot (P_{P,k} + P_{PQE,k}) - C_{PQE,k}) \tag{29}$$

- *CPP* with back-pressure turbines:
  to minimize the cost

$$min \sum_{l=1}^{n4} (C_{PQBP,l} - h_Q \cdot Q_{PQBP,l} - h_P \cdot (P_{PQBP,l})) \tag{30}$$

or maximizing the profit

$$max \sum_{l=1}^{n4} (h_Q \cdot Q_{PQBP,l} + h_P \cdot (P_{PQBP,l}) - C_{PQBP,l}) \tag{31}$$

Solution Methods

The optimal dispatch problem of the cogeneration power system can be solved by the same methods that have been used for economic dispatching of the electric power systems [1].

Here the λ-iteration method is used. Figure 2 presents the outline of the proposed method.

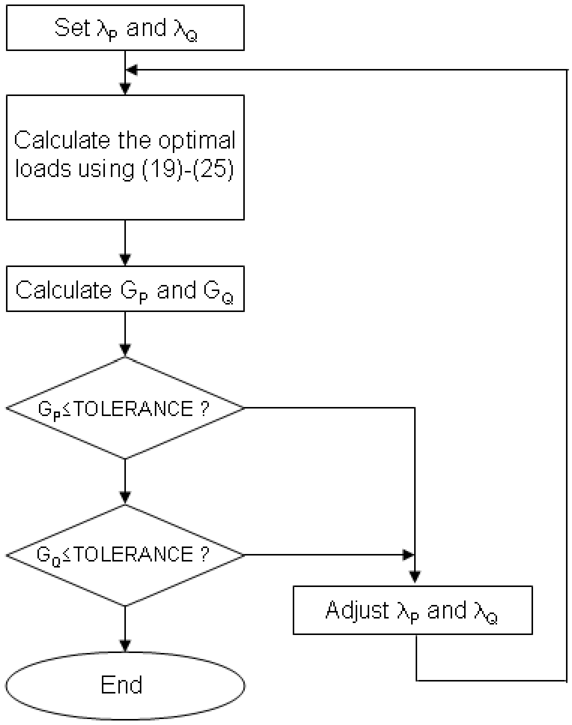

Fig. 2. The outline of the proposed method

Practical Results

A computer program for optimal load dispatch and unit commitment of cogeneration system is developed. The program is composed in Watcom Fortran 10.6 and C languages. Database is in FoxPro.

Conclusions

There are four general possibilities to optimize cogeneration units or plants in the cogeneration system:

1) complex optimization of operation on the cogeneration system level;
2) optimization of CPP only on the district heating system level;
3) optimization of CPP only on the power system level;
4) decentralised optimization of CPP.

The method presented in the paper enables optimizing of CPP without distribution of the cost of fuel between heat and electric power loads. The costs for optimization of heat load are taken equal to the total cost of fuel minus the income from selling electrical energy, and the costs for optimization of electrical loads are taken equal to the total cost of fuel minus the income from selling heat energy.

The optimal dispatch problem is interrelated with unit commitment and incompleteness of information (with taking into account emission taxes and also probabilistic, uncertain and fuzzy information [6–8]).

The unit commitment problem is much more complicated. An efficient method for the solution of this problem is the forward dynamic programming [1, 9]. The objective function consists of fuel cost, start-up cost, shutdown cost, auxiliary cost and income from electricity sales. This problem has numerous constraints and limits.

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