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Radiation FIELD IN AN ATMOSPHERE SUBJECTED TO COSINE VARYING DIFFUSE RADIATION

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Abstract. The radiation field is calculated in an optically finite or semi-infinite, two-dimensional, plane-parallel, absorbing–emitting but nonscattering grey atmosphere subjected to diffuse cosine varying incident boundary radiation. We again approximate the kernel of the integral equation for the emissive power by a sum of exponents. After this approximation the integral equation can be solved exactly. The solution can be written in generalized x- and y-functions (or h- and g-functions in the semi-infinite case) which were introduced for a one-dimensional atmosphere. Since the radiation field in the case of diffuse incident radiation can be described as a superposition of solutions for the collimated case, we can find the accurate values for the source function and the radiative flux at arbitrary optical depths in the atmosphere. As in the case of collimated incident radiation, this approximation allowed of finding accurate numerical values for the source function and the radiative flux.

Key words: two-dimensional radiative transfer, X- and Y-functions, emissive power, radiative flux.

1. INTRODUCTION

In previous papers [1,2] we studied the radiative transfer in two-dimensional, optically semi-infinite and finite atmospheres subjected to collimated cosine radiation. By applying the kernel approximation method to a simplified integral equation for the source function (or the temperature distribution, or the emissive power) it was possible to find the radiative field at any point in the atmosphere. In this paper these results are generalized for an optically semi-infinite or finite atmosphere, allowing for diffuse cosine varying radiation incident on one (or both) of its boundaries. Breig and Crosbie who have found the external radiation field for these atmospheres [3–5] have already stressed that the cosine boundary conditions are not physically realistic. At the same time they are useful since the solutions for other, more realistic problems can be expressed in terms of the cosine solutions (Fourier theorem!). Perhaps diffuse incident radiation can serve more realistically to solve some problems, e.g. in astronomy and atmospheric physics.

2. EQUATION OF RADIATIVE TRANSFER

We are looking for the emissive power (or the temperature distribution, or the source function) in a homogeneous nonscattering, plane-parallel, two-dimensional grey atmosphere which is in local thermodynamic equilibrium. The radiation field in such an atmosphere is described by the following equation [6]

$$\cos\theta \frac{\partial I(\tau_y,\tau_z,\theta,\phi,\tau_0)}{\partial \tau_z} + \sin\theta\sin\phi \frac{\partial I(\tau_y,\tau_z,\theta,\phi,\tau_0)}{\partial \tau_y}$$
$$= -I(\tau_y,\tau_z,\theta,\phi,\tau_0) + S(\tau_y,\tau_z,\theta,\phi,\tau_0), \tag{1}$$

where I is the intensity; θ, the polar angle measured from the inward normal to the atmosphere; ϕ, the azimuthal angle measured from the τ_x-axis; S, the emissive power in the atmosphere (this defines also the temperature distribution in the atmosphere and quite often this function is called the source function). The optical depth τ_z is measured downward from the boundary of the atmosphere and it forms together with τ_x and τ_y a right-hand rectangular co-ordinate system. In the atmosphere the energy is transferred only by radiation, i.e. there is no heat conduction or convection.

We apply the integrating factor techniques to Eq. (1) and as a result obtain the formal solution for the intensities of downward and upward moving radiation in the form

$$I^+(\tau_y,\tau_z,\mu)$$
$$= I_0(\tau_y^+)\exp(-\tau_z/\mu) + \frac{1}{\pi}\int_0^{\tau_z} S(\tau_y',\tau_z',\tau_0)\exp(-(\tau_z-\tau_z')/\mu)\mathrm{d}\tau_z'/\mu \tag{2}$$

and

$$I^-(\tau_y,\tau_z,\mu) = \frac{1}{\pi}\int_{\tau_z}^{\tau_0} S(\tau_y',\tau_z',\tau_0)\exp(-(\tau_z'-\tau_z)/|\mu|)\mathrm{d}\tau_z'/\mu, \tag{3}$$

where

$$\begin{aligned} \tau_y^+ &= \tau_y - \tau_z\tan\theta\sin\phi, \\ \tau_y' &= \tau_y + (\tau_z'-\tau_z)\tan\theta\sin\phi, \end{aligned} \tag{4}$$

and $\mu = \cos\theta$, τ_0 is the optical thickness of the atmosphere in z-direction and I_0^+ is the intensity incident on the upper boundary of the atmosphere [6].

This very complicated change of variables, done by Smith in [7], reduces the corresponding two-dimensional integral equation to a one-dimensional integral equation.

As we require the atmosphere to be in radiative equilibrium, we can write

$$S(\tau_y, \tau_z) = \int_{4\pi} I \mathrm{d}\omega, \tag{5}$$

where ω is the solid angle and $\mathrm{d}\omega$ $=\mathrm{d}\mu\mathrm{d}\phi$. Substituting Eqs. (2) and (3) into Eq. (5), we obtain the equation for the emissive power

$$\begin{aligned} S(\tau_y, \tau_z, \tau_0) &= \int_{2\pi} I_0^+(\tau_y^+) \exp(-\tau_z/\mu)\mathrm{d}\omega \\ &+\frac{1}{\pi}\int_0^{2\pi}\int_0^1\int_0^{\tau_0} S(\tau_y', \tau_z', \tau_0)\exp(-\left|\tau_z - \tau_z'\right|/\mu)\mathrm{d}\tau_z'\mathrm{d}\mu/\mu\mathrm{d}\phi. \end{aligned} \tag{6}$$

In the following we need also the z-component of the radiative flux which can be written in the form

$$q_z(\tau_y, \tau_z, \tau_0) = \int_{4\pi} I(\tau_y, \tau_z, \theta, \phi, \tau_0)\mu\mathrm{d}\omega, \tag{7}$$

or, taking into account Eqs. (2) and (3),

$$\begin{aligned} q_z(\tau_y,\tau_z, \tau_0) = \int_0^{2\pi}\int_0^1 \Big[& I^+(\tau_y^+, 0, \mu, \phi)\exp(-\tau_z/\mu) \\ &+ \int_0^{\tau_0} S(\tau_y', \tau_z', \tau_0)\mathrm{sign}(\tau_z - \tau_z')\exp(-\left|\tau_z - \tau_z'\right|/\mu)\mathrm{d}\tau_z'/\mu\Big]\mu\mathrm{d}\mu\mathrm{d}\phi. \end{aligned} \tag{8}$$

Now we assume that there is incident radiation only on the surface $\tau_z = 0$ and that it is cosine varying but diffuse, i.e. incident radiation does not depend on the direction of incidence. In this case the boundary condition for $\tau_z = 0$ and $0 \leq \mu \leq 1$ is expressed as

$$I^+(\tau_y^+, 0, \theta, \phi) = I_0^+\left[1 + \epsilon\cos(\beta\tau_y^+)\right], \tag{9}$$

and for $\tau_z = \tau_0$ and $-1 \leq \mu \leq 0$

$$I^-(\tau_y^-, \tau_0, \theta, \phi) = 0, \tag{10}$$

where

$$\tau_y^- = \tau_y + (\tau_0 - \tau_z)\tan\theta\sin\phi.$$

In Eq. (9), I_0 is a constant, ϵ is the amplitude of the cosine wave, and β is the spatial frequency of the strips of illumination. In other words, the top of the atmosphere is illuminated stripwise by diffuse rays, while the strips are parallel to the x-axis and their width is defined by the spatial frequency β. The wave pattern of illumination repeats itself along the τ_y-axis while the spatial period is π/β. To solve Eq. (6), we derive a one-dimensional integral equation for the emissive power by applying the concept of separation of variables [4] in the form

$$S(\tau_y, \tau_z, \tau_0) = S_0 \left[J_{\beta=0}(\tau_z, \tau_0) + \epsilon J_\beta \left(\tau_z, \tau_0\right) \cos(\beta \tau_y) \right], \tag{11}$$

where the dimensionless emissive power J_β can be shown to satisfy the integral equation

$$J_\beta \left(\tau_z, \tau_0\right) = \frac{1}{2} \mathcal{E}_2 \left(\tau_z, \beta\right) + \frac{1}{2} \int_0^{\tau_0} \mathcal{E}_1 \left(\tau_z - \tau_z', \beta\right) J_\beta \left(\tau_z', \tau_0\right) \mathrm{d}\tau_z', \tag{12}$$

with the generalized exponential integrals of the first and second order defined by

$$\mathcal{E}_1 \left(\tau, \beta\right) = \int_0^p \frac{\exp\left(-\left|\tau\right|/t\right)}{\sqrt{1 - \beta^2 t^2}} \frac{\mathrm{d}t}{t} \tag{13}$$

and

$$\mathcal{E}_2 \left(\tau, \beta\right) = \int_0^p \frac{\exp(-\left|\tau\right|/t)}{\left(1 - \beta^2 t^2\right)^{3/2}} \mathrm{d}t, \tag{14}$$

while $p = \left(1 + \beta^2\right)^{-1/2}$.

We solve Eq. (12), both for the cases $\beta = 0$ and $\beta \neq 0$, by using the approximation of the Sobolev resolvent function [8], described in [9].

Using the dimensionless emissive power for the cosine varying collimated boundary condition, we can easily find the respective emissive power for the cosine varying diffuse boundary condition [3]. We start from the dimensionless emissive power for the cosine varying collimated boundary condition. In this case the dimensionless emissive power satisfies the following integral equation [3]

$$J_\beta^{\mathrm{col}} \left(\tau_z, \mu_0, \tau_0\right) = \exp\left(-\tau_z/\mu_0\right) + \frac{1}{2} \int_0^{\tau_0} \mathcal{E}_1 \left(\tau_z - \tau_z', \beta\right) J_\beta^{\mathrm{col}} \left(\tau_z', \mu_0, \tau_0\right) \mathrm{d}\tau_z'. \tag{15}$$

Next we multiply Eq. (15) by the function

$$\psi_1(\mu_0, \beta) = \left(1 - \beta^2 \mu_0^2\right)^{-3/2} \tag{16}$$

and integrate the result from 0 to p

$$\int_0^p J_\beta^{\mathrm{col}} \left(\tau_z, \mu_0, \tau_0\right) \psi_1(\mu_0, \beta) \mathrm{d}\mu_0 = \mathcal{E}_2(\tau, \beta) + \frac{1}{2} \int_0^{\tau_0} \mathcal{E}_1 \left(\tau_z - \tau_z', \beta\right) \int_0^p J_\beta^{\mathrm{col}} \left(\tau_z', \mu_0, \tau_0\right) \psi_1(\mu_0, \beta) \mathrm{d}\mu_0 \mathrm{d}\tau_z'. \tag{17}$$

Comparing Eqs. (15) and (17) we see that the forcing function of Eq. (17) is a superposition of the forcing function of Eq. (15). Next we take into account the superposition principle – if we have two linear Fredholm integral equations with equal kernels and the forcing function of the second equation is a superposition of the forcing function of the first equation, then the solution of the second equation is also the superposition of the solution of the first equation. Consequently,

$$J_\beta\left(\tau_z,\tau_0\right)=\frac{1}{2}\int_0^p J_\beta^{\mathrm{col}}\left(\tau_z,u,\tau_0\right)\psi_1(u,\beta)\mathrm{d}u, \tag{18}$$

or, in our notations,

$$\begin{aligned} &J_\beta(\tau,\tau_0)\\ &=\frac{1}{2}\int_0^p \psi_1(u,\beta)\left\{X_\beta(u,\tau_0)y(\tau,u,\tau_0)-Y_\beta(u,\tau_0)\left[x_\beta(\tau_0-\tau,u,\tau_0)-1\right]\right\}\mathrm{d}u. \end{aligned} \tag{19}$$

The functions $X_\beta(u,\tau_0)$, $Y_\beta(u,\tau_0)$, $y(\tau,u,\tau_0)$, and $x_\beta(\tau,u,\tau_0)$ are defined[1], and their determination is described, in [[2]]. In the same manner we can find the radiative flux. According to Breig and Crosbie [[4]], we can write

$$q_z(\tau_y,\tau_z,\tau_0)=\pi I_0^+[F_{\beta=0}(\tau_z,\tau_0)+\epsilon F_\beta(\tau_z,\tau_0)\cos(\beta\tau_y)], \tag{20}$$

where F_β is the dimensionless flux

$$F_\beta(\tau_z,\tau_0)=2\mathcal{E}_3\left(\tau_z,\beta\right)+2\int_0^{\tau_0} J_\beta^{\mathrm{col}}\left(\tau_z',\tau_0\right)\mathrm{sign}(\tau_z-\tau_z')\mathcal{E}_2\left(\tau_z-\tau_z',\beta\right)\mathrm{d}\tau_z' \tag{21}$$

and $\mathcal{E}_3$ is the generalized exponential integral of order three, i.e.

$$\mathcal{E}_3\left(\tau_z,\beta\right)=\tau\int_1^\infty \mathcal{E}_2\left(\tau_z t,\beta/t\right)\mathrm{d}t. \tag{22}$$

Using the results of Breig and Crosbie [[3]], we can write Eq. (21) in the form

$$F_\beta(\tau_z,\tau_0)=2\mathcal{E}_3\left(\tau_z,\beta\right)+2\int_0^p F_\beta^{\mathrm{col}}\left(\tau_z,u,\tau_0\right)\psi_1(\beta,u)\mathrm{d}u-2\mathcal{B}(\tau,\beta), \tag{23}$$

where $F_\beta^{\mathrm{col}}\left(\tau_z,u,\tau_0\right)$ is given by Eq. (34) in [[2]] and

$$\mathcal{B}(\tau,\beta)=\int_0^p u\exp\left(-\tau_z/u\right)\psi_1(\beta,u)\mathrm{d}u. \tag{24}$$

[1] Typographical errors have crept into Eqs. (18)–(21) in [[2]]. The summation starts from $k=1$ in these equations. In Eqs. (18) and (20) there must be a summand 1, in Eq. (19) a summand $\exp(-\tau/\mu_0)$, and in Eq. (21) a summand $\exp(-\tau_0/\mu_0)$, outside the summation sign.

The z-component of the diffuse flux at the boundaries is given by Breig and Crosbie [3]

$$F_\beta(0,\tau_0) = 1 + 2\int_0^p F_\beta^{\mathrm{col}}(0,u,\tau_0)\,\psi_1(\beta,u)\mathrm{d}u - \frac{2}{\beta^2}\left(\sqrt{1+\beta^2}-1\right), \quad (25)$$

since [3] $\mathcal{E}_3(0) = 1/2$, and

$$F_\beta(\tau_0,\tau_0) = 2\mathcal{E}_3(\tau_0,\beta) + 2\int_0^p F_\beta^{\mathrm{col}}(\tau_0,u,\tau_0)\,\psi_1(\beta,u)\mathrm{d}u - 2\mathcal{B}(\tau_0,\beta). \quad (26)$$

This completes the solution of the posed problem.

3. NUMERICAL RESULTS

In order to get numerical results we used the quadrature scheme outlined in [1]: we divided the integration range into four subintervals $(0, 0.9p), (0.9p, 0.99p), (0.99p, 0.999p)$, and $(0.999p, p)$ and in each subinterval we used Gautschi's rule [10] with the weight function given by Eq. (16). The order of quadrature $N = 84$ gave at least five accurate significant figures for the source function when summing in Eq. (19).

The calculation of the flux was much more complicated. The integral term in Eq. (23) could be tackled in the same way as in Eq. (19). As already pointed out by Breig and Crosbie in [3], finding numerical values for the functions $\mathcal{E}_3(\tau,\beta)$ and $\mathcal{B}(\tau,\beta)$ "presents an added numerical difficulty".

First of all, the definition for the function $\mathcal{E}_3$ must be redefined in the form [11]

$$\mathcal{E}_3(\tau,\beta) = \frac{1}{2}\tau\left[\mathcal{V}(\tau,\beta) - \mathcal{E}_2(\tau,\beta)\right], \quad (27)$$

where

$$\mathcal{V}(\tau,\beta) = \int_1^\infty \exp(-\tau(t^2+\beta^2)^{1/2})\mathrm{d}t. \quad (28)$$

Next we change the variables in Eq. (14), Eq. (24), and Eq. (28) and obtain formulas for $\mathcal{E}_3$ and $\mathcal{B}$ in the form

$$\mathcal{B}(\tau,\beta) = \sqrt{1+\beta^2}\int_0^1 \frac{x}{\left[1+\beta^2(1+x^2)\right]^{3/2}}\exp\left(-\tau\sqrt{1+\beta^2}/x\right)\mathrm{d}x, \quad (29)$$

$$\mathcal{E}_3(\tau,\beta) = \frac{1}{2}\tau(1+\beta^2)^2\int_0^1 \frac{1-x^2}{x^2\left[1+\beta^2(1-x^2)\right]^{3/2}}\exp\left(-\tau\sqrt{1+\beta^2}/x\right)\mathrm{d}x. \quad (30)$$

The accurate numerical values of the $\mathcal{B}$-function in Eq. (29) for a broad region of both τ and β could easily be found by using either Gautschi's scheme [10] or a very powerful code DQAG by Piessens and de Doncker [12]. Trying to numerically evaluate the $\mathcal{E}_3$-function, we ran into trouble since neither of these schemes could ensure the needed accuracy ($\epsilon = 10^{-7}$). When looking for the best possible quadrature formula for our problem, we came across the second Euler–Maclaurin summation code DMIDPNT in [13]. This work-horse, as the authors called it, was able to give very accurate results, at the expense of much longer computing time, though. For checking purposes we used MAPLE in the SWP package [14].

Figure 1 shows the behaviour of the source function $S(\tau_y, \tau_z)$ for $\beta = 1.0$ in a semi-infinite atmosphere. When we compare this with the source function in Fig. 2, where $\beta = 10.0$, we notice that in the case of a larger spatial frequency β the asymptotic regime is reached at much smaller optical depths.

For atmospheres with finite optical thicknesses the impact of incident diffuse cosine radiation with smaller spatial frequencies on the source function can be observed even at the bottom of the atmosphere, while that of the larger spatial frequencies dies off before reaching the bottom (Figs. 3 and 4).

What has been said about the behaviour of the source function is valid also in the case of the flux (Figs. 5–8).

We compared the respective parameters of the radiation field in the cases of diffuse incident radiation and collimated incident radiation. While for the diffuse case the parameters were monotonic functions of the optical depth τ_z, this behaviour was lost for the collimated case.

4. CONCLUSION

The use of the superposition principle in solving the integral equations for the source function allows us to consider the source function in the case of diffuse incident radiation as a superposition of solutions for collimated incident radiation. This means that the method used for finding the radiation field in an atmosphere subjected to collimated incident radiation can be used as a basic method to solve for the radiation field in a broad range of incident radiation patterns.

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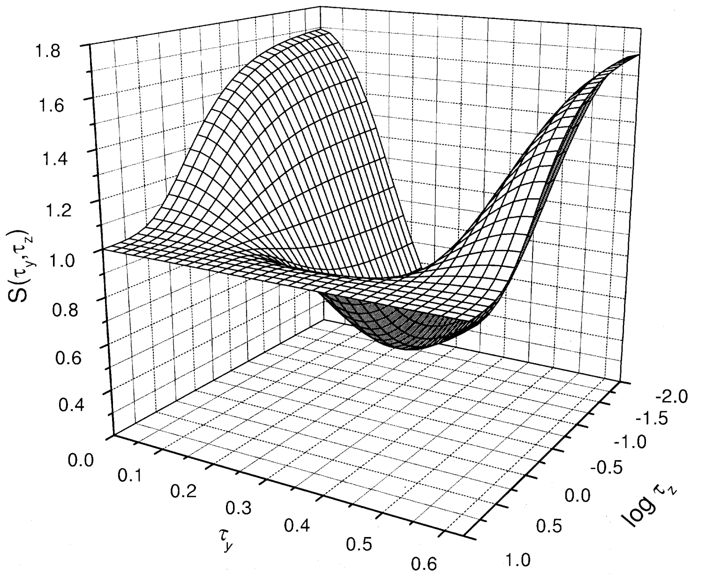

Fig. 1. The source function S as a function of optical depths τ_y and τ_z in an optically semi-infinite atmosphere. The spatial frequency β = 1.0.

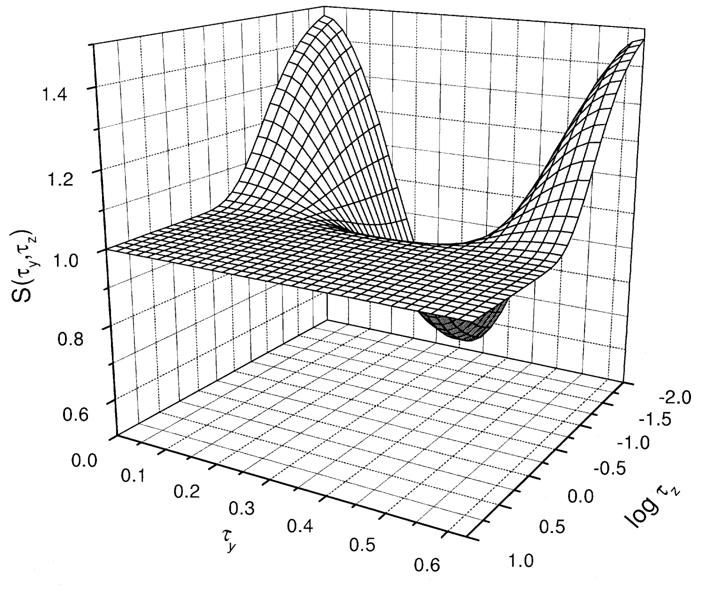

Fig. 2. Same as Fig. 1, only the spatial frequency $\beta = 10.0$.

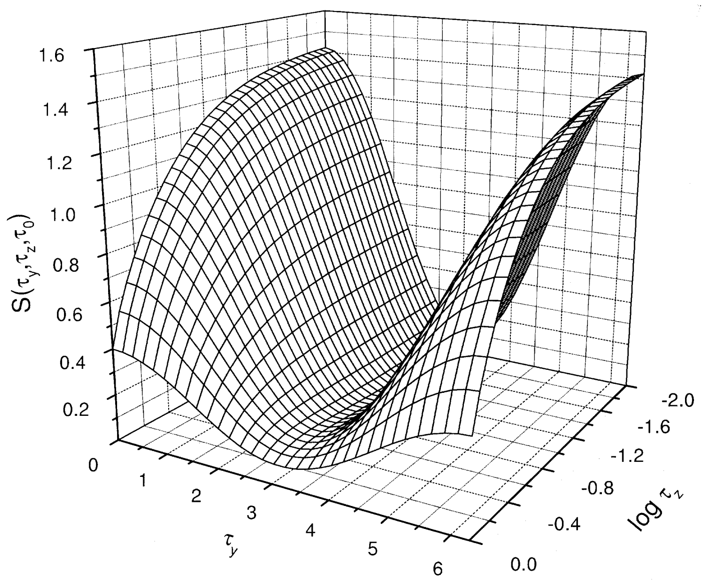

Fig. 3. Same as Fig. 1, only the optical thickness of the atmosphere $\tau_0 = 1.0$.

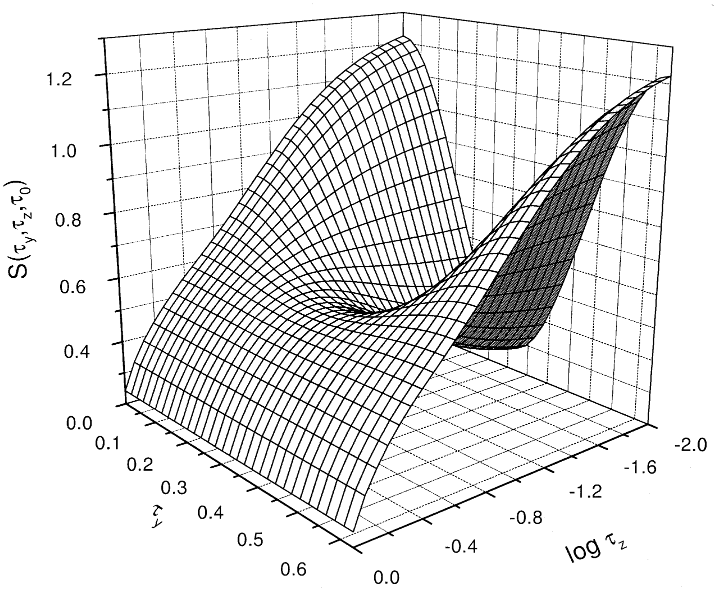

Fig. 4. Same as Fig. 1, only the spatial frequency $\beta = 10.0$ and the optical thickness of the atmosphere $\tau_0 = 1.0$.

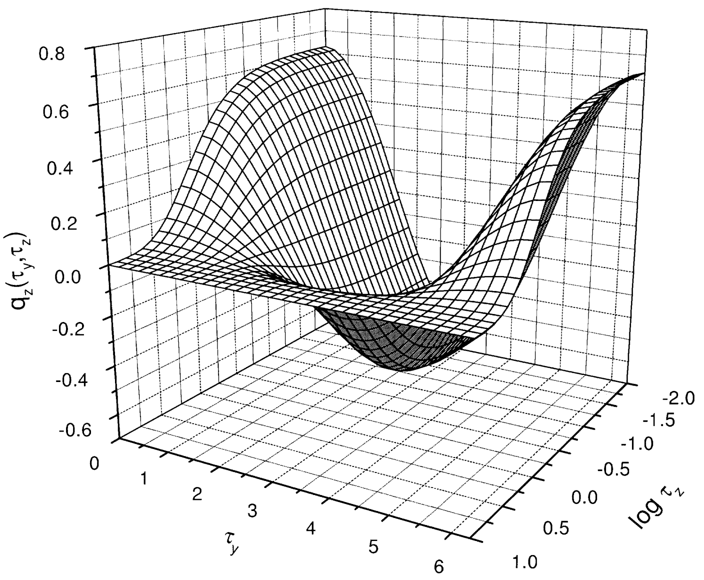

Fig. 5. The radiative flux q_z as a function of optical depths τ_y and τ_z in an optically semi-infinite atmosphere. The spatial frequency $\beta = 1.0$.

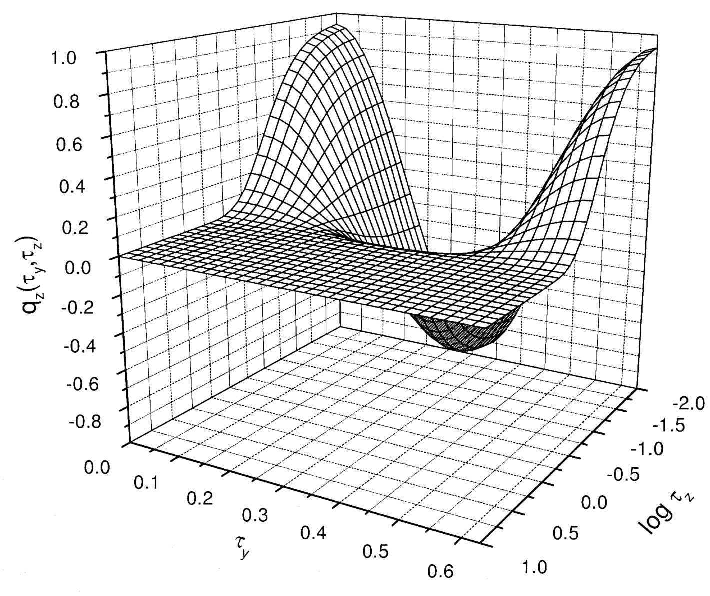

Fig. 6. Same as Fig. 5, only the spatial frequency $\beta = 10.0$.

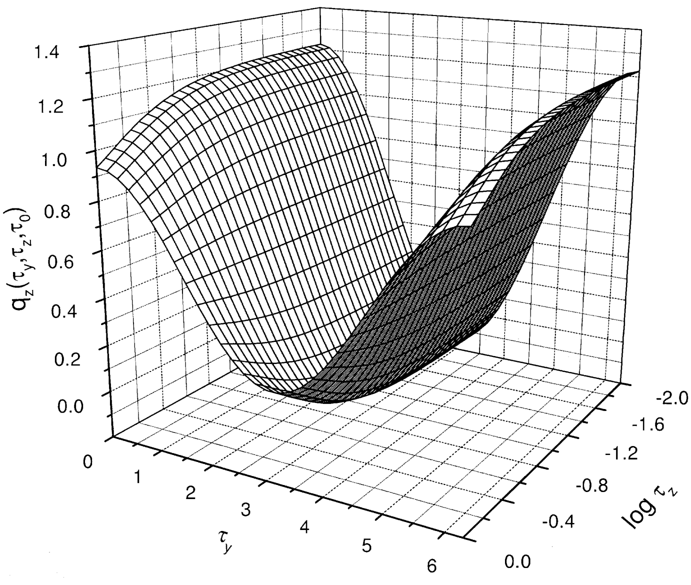

Fig. 7. Same as Fig. 5, only the optical thickness of the atmosphere $\tau_0 = 1.0$.

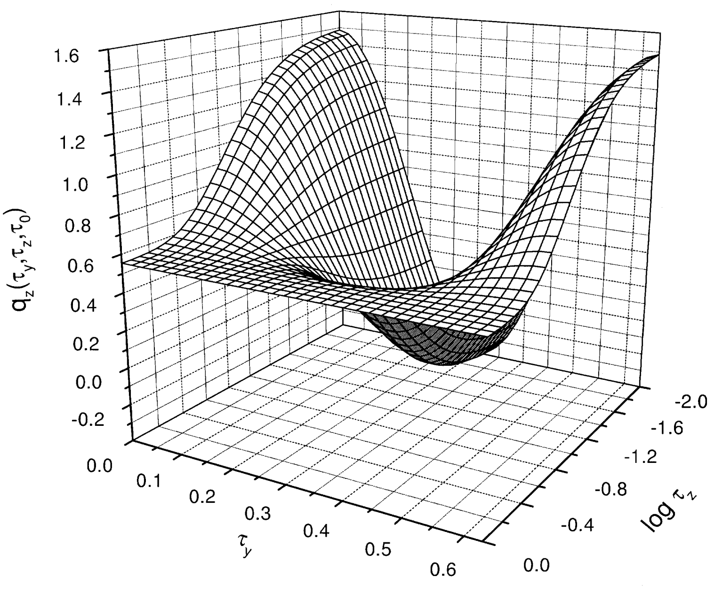

Fig. 8. Same as Fig. 5, only the spatial frequency $\beta = 10.0$ and the optical thickness of the atmosphere $\tau_0 = 1.0$.

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KIIRGUSVÄLI ATMOSFÄÄRIS, MILLELE LANGEB KOOSINUSSEADUSE JÄRGI MUUTUV DIFUUSNE KIIRGUS

Tõnu VIIK ja Indrek VURM

Vaadeldi kiirguslevi optiliselt lõpliku ja optiliselt poollõpmatu paksusega kahemõõtmelises tasaparalleelses mittehajutavas, kuid neelavas ja kiirgavas atmosfääris, millele langeb koosinusseaduse järgi muutuv difuusne kiirgus. Samuti kui eelmistes artiklites [1,2] oletatakse, et atmosfäär on hall ning ta on kiirguslikus ja lokaalses termodünaamilises tasakaalus. Praegusel juhul saab kiirguslevi võrrandi taandada integraalvõrrandiks, mille omakorda saab muutujate eraldamise teel

taandada suhteliselt lihtsaks integraalvõrrandiks ühemõõtmelise keskkonna kohta, kui oletame, et pealelangeva kiirguse omadused x-telje suunas ei muutu. Difuusse pealelangeva kiirguse puhul allikfunktsiooni jaoks leitud integraalvõrrandi vabaliige on kollimeeritud pealelangeva kiirguse puhul leitud analoogilise võrrandi vabaliikmete superpositsioon. Järelikult on ka difuusse pealelangeva kiirguse puhul integraalvõrrandi lahendiks vastavate kollimeeritud juhu lahendite superpositsioon. Seega saab lahendi lihtsalt leida, arvutuslikke raskusi valmistasid vaid kiirgusvoo avaldisse ilmuvad integraalid. Neid õnnestus leida vajaliku täpsusega ($\epsilon = 10^{-7}$) Euleri–Maclaurini teise summeerimisvalemi abil. Kontrolliks kasutasime Scientific WorkPlace'i integreeritud programmipaketti MAPLE. Artiklis oleme vaadelnud kiirgusvälja kahe olulise parameetri – allikfunktsiooni ja kiirgusvoo – käitumist lõpliku ja poollõpmatu optilise paksusega atmosfääris pealelangeva kiirguse triipude erinevate laiuste puhul. Selgus, et poollõpmatus atmosfääris kitsamate triipude (suuremate β väärtuste) puhul jõudis allikfunktsioon asümptootsesse režiimi palju väiksematel optilistel sügavustel kui laiemate triipude puhul. Lõpliku optilise paksusega atmosfäärides ulatus laiemate triipude mõju atmosfääri alumise pinnani välja, kuna kitsamate triipude puhul pealelangeva kiirguse mõju sumbus kiiresti. Kõik eelöeldu kehtib ka kiirgusvoo kohta.

Nii allikfunktsioon kui kiirgusvoog difuusse pealelangeva kiirguse puhul on optilise sügavuse τ_z suhtes monotoonsed funktsioonid, mida ei saa aga alati öelda kollimeeritud pealelangeva kiirguse puhul.