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CONTROL THEORY

A note on the relationship between single- and multi-experiment observability for discrete-time nonlinear control systems

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Abstract. The connection between the concepts of the single-experiment and the multi-experiment unobservability of a nonlinear discrete-time control system is studied. The main result claims that if the system is single-experiment unobservable and the observable space is integrable, then the system is also multi-experiment unobservable. For the proof of the main result a novel mathematical technique, the so-called algebra of functions, is used.

Key words: nonlinear control system, observability, differential one-form, algebra of functions.

1. INTRODUCTION

Observability is a fundamental property of the control system. Different applications rely on different observability concepts. In some situations, one is given just a single sequence of inputs and the corresponding sequence of outputs to find the arbitrary initial state. If this is possible, the system is called single-experiment observable. In some other cases, one is allowed to use several input sequences together with the corresponding output sequences to determine the initial state. The associated observability concept is called multi-experiment observability. Obviously, if the system is single-experiment observable, it is also multi-experiment observable. For continuous-time systems also the converse is proved to hold, at least for analytic systems [5]. In the discrete-time case, for the converse to hold, one has to assume additionally that the (analytic) system is reversible [5]. However, for discrete-time nonlinear control systems, simple bilinear nonreversible examples exist, demonstrating that the system may be single-experiment unobservable but multi-experiment observable [2] (see also a more complicated example in [5]). The goal of this short paper is to study further the connection between two observability notions, and relate it with the integrability of the observable space. Note that the possible non-integrability of the observable space is purely a discrete-time phenomenon [2,3] like the possibility that the analytic system may be multi-experiment observable but not single-experiment observable. The main result of this paper claims that if the system is single-experiment unobservable and the observable space is integrable, then the system is also multi-experiment unobservable. No reversibility assumption is made in proving this result.

2. PRELIMINARIES

Consider a single-input single-output nonlinear discrete-time control system, described by the state equations

$$\begin{aligned} x^{+} &= f(x,u), \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in X \subset \mathbb{R}^n$ is the state variable, $u \in U \subset \mathbb{R}$ is the input variable, $y \in Y \subset \mathbb{R}$ is the output variable, $f : \mathbb{R}^n \times U \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to Y$ are the real-analytic functions. Notice that in this paper we use symbols $^+$ and $^{[i]}$ instead of the arguments $t+1$ and $t+i$, respectively, to

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simplify the exposition, so $x^+ := x(t+1)$, $x := x(t)$, and $y^{[i]} = y(t+i)$.

Assume that the map $(x,u) \mapsto f(x,u)$ generically defines a submersion, i.e. generically

$$\operatorname{rank} \frac{\partial f(x,u)}{\partial (x,u)} = n \quad (2)$$

holds. Let $\mathscr{K}$ denote the field of meromorphic functions in a finite number of independent system variables $\{x(0), u(t), t \geq 0\}$. The forward-shift operator $\delta : \mathscr{K} \to \mathscr{K}$ is defined by $\delta \psi(x,u) = \psi(f(x,u), u^+)$. Over the field $\mathscr{K}$ one can define a difference vector space $\mathscr{E} := \operatorname{span}_{\mathscr{K}}\{\mathrm{d}\varphi \mid \varphi \in \mathscr{K}\}$. The operator δ induces a forward-shift operator $\delta : \mathscr{E} \to \mathscr{E}$ by

$$\sum_i a_i \mathrm{d}\varphi_i \to \sum_i \delta a_i \mathrm{d}(\delta \varphi_i), \quad a_i, \varphi_i \in \mathscr{K}.$$

Under the assumption (2), $\mathscr{K}$ is a difference field [2].

2.1. Observability

Both in the single- or multi-experiment context, the observability property that is easiest to characterize for nonlinear systems is local weak observability. In the rest of the paper, local weak observability will simply be called observability. Further discussion for discrete-time systems can be found in [1].

Given a system of the form (1), let us denote by $\mathscr{X}$, $\mathscr{Y}^k$, $\mathscr{Y}$, and $\mathscr{U}$ the following subspaces of differential one-forms:

$$\begin{aligned} \mathscr{X} &:= \operatorname{span}_{\mathscr{K}}\{\mathrm{d}x\}, \\ \mathscr{Y}^k &:= \operatorname{span}_{\mathscr{K}}\{\mathrm{d}y^{[j]}, 0 \leq j \leq k\}, \\ \mathscr{Y} &:= \operatorname{span}_{\mathscr{K}}\{\mathrm{d}y^{[j]}, j \geq 0\}, \\ \mathscr{U} &:= \operatorname{span}_{\mathscr{K}}\{\mathrm{d}u^{[j]}, j \geq 0\}. \end{aligned}$$

The chain of subspaces

$$0 \subset \mathscr{O}_0 \subset \mathscr{O}_1 \subset \mathscr{O}_2 \subset \ldots \subset \mathscr{O}_k \subset \ldots,$$

where $\mathscr{O}_k := \mathscr{X} \cap \left(\mathscr{Y}^k + \mathscr{U}\right)$ is called the *observability filtration*. If we denote by $\mathscr{O}_\infty$ the limit of the observability filtration, it is easy to see that

$$\mathscr{O}_\infty = \mathscr{X} \cap (\mathscr{Y} + \mathscr{U})$$

and we can introduce the following definition.

Definition 1. *The subspace $\mathscr{O}_\infty$ is called the observable space of system* (1).

Proposition 1 ([2]). *System* (1) *is locally single-experiment observable if $\mathscr{O}_\infty = \mathscr{X}$.*

If the observable space is integrable, the system can be decomposed into the observable and unobservable subsystems. Unfortunately, unlike in the continuous-time case for discrete-time systems, $\mathscr{O}_\infty$ is not necessarily integrable [2]. If $\mathscr{O}_\infty$ is integrable, and therefore, has locally an exact basis $\{\mathrm{d}z_1, \ldots, \mathrm{d}z_r\}$, one can complete the set $\{z_1, \ldots, z_r\}$ to a basis $\{z_1, \ldots, z_r, z_{r+1}, \ldots, z_n\}$ of $\mathscr{X}$, where $z_1, \ldots, z_r$ are observable coordinates and $z_{r+1}, \ldots, z_n$ are unobservable coordinates. Then, in these coordinates, the system can be decomposed into observable and unobservable subsystems

$$\begin{aligned} z_1^+ &= f_1(z_1, \ldots, z_r, u), \\ &\vdots \\ z_r^+ &= f_r(z_1, \ldots, z_r, u), \\ z_{r+1}^+ &= f_{r+1}(z, u), \\ &\vdots \\ z_n^+ &= f_n(z, u), \\ y &= h(z_1, \ldots, z_r). \end{aligned}$$

For each control sequence $\omega \in \mathbb{R}^k$, define $f^\omega : X \to X$ inductively by $f^e(x) = x$ for the empty sequence and $f^{\omega u}(x) = f(f^\omega(x), u)$. We also let $h^\omega := h \circ f^\omega$. For $\mu = (\mu_0, \mu_1, \ldots) \in \mathbb{R}^\infty$, let $H^\mu(x) := (h(x), h^{\mu_0}(x), h^{\mu_0\mu_1}(x), \ldots)$. Two states x^1 and x^2 are said to be distinguished by an input sequence $\mu \in \mathbb{R}^\infty$ if $H^\mu(x^1) \neq H^\mu(x^2)$ [5].

Definition 2. *System* (1) *is said to be multi-experiment observable if any two distinct states x^1, x^2 can be distinguished by some input sequence μ.*

To prove the main result of the paper, we apply a special mathematical technique, the so-called algebra of functions.

2.2. Algebra of functions

The main elements of the algebra of functions are binary relations, operations, and operators. Here those concepts, necessary to understand this paper, are recalled; for more details see [4,6,7].

Let $X \subseteq \mathbb{R}^n$ be a vector space. Denote by $\mathfrak{I}_X$ the set of vector functions with the domain X. Consider two arbitrary functions $\alpha : X \to S$ and $\beta : X \to W$ from $\mathfrak{I}_X$, where $S \subseteq \mathbb{R}^s$ and $W \subseteq \mathbb{R}^r$ are some vector spaces. One can define the binary relation of partial preorder for functions α and β as follows.

Definition 3. *We say that α is less than or equal to β and denote this as $\alpha \leq \beta$ if there exists a function γ such that the composition of α with γ, $\gamma(\alpha(x)) = \beta(x), \forall x \in X$.*

Definition 4. *The functions $\alpha, \beta \in \mathfrak{I}_X$ are equivalent, and denoted by $\alpha \cong \beta$, if both inequalities $\alpha \leq \beta$ and $\beta \leq \alpha$ hold.*

Definition 5. *Given $\alpha, \beta \in \mathfrak{I}_X$,*

$$\alpha \times \beta = \max(\gamma \in \mathfrak{I}_X \mid \gamma \leq \alpha, \gamma \leq \beta).$$

The rule of operation $\times$ is simple

$$(\alpha \times \beta)(x) = \begin{bmatrix} \alpha(x) \\ \beta(x) \end{bmatrix}.$$

Definition 6. *We say that the functions $\alpha, \beta \in \Im_X$ form a pair and denote this as $(\alpha, \beta) \in \Delta$ if there exists a function f^* such that $\beta(f(x,u)) = f^*(\alpha(x), u)$ for every $(x,u) \in X \times U$.*

Definition 7. *The function $\alpha \in \Im_X$ is called* f-invariant *if $(\alpha, \alpha) \in \Delta$, i.e if there exists a function $f^* : S \times U \to S$ such that $\alpha(f(x,u)) = f^*(\alpha(x), u)$ for every $(x,u) \in X \times U$.*

Definition 8. *The function $\alpha \in \Im_X$ is called a* maximal f-invariant function *if for every f-invariant function α^* the following holds: $\alpha^* \leq \alpha$.*

Note that Definitions 3–8 hold sometimes only locally. In such a case all claims hold for some open and dense subset of X rather than for X. Consider for instance the following example[1].

Example 1. Consider the functions $\alpha = \arctan\left(\frac{x_1}{x_2}\right)$ and $\beta = \arctan\left(\frac{x_2}{x_1}\right)$. To verify whether the functions are equivalent or not, first we show that

$$\alpha \cong \frac{x_1}{x_2}. \tag{3}$$

Since there exists a function γ such that $\gamma\left(\frac{x_1}{x_2}\right) = \alpha$, according to Definition 3, $\frac{x_1}{x_2} \leq \alpha$. Moreover, there exist $\bar{\gamma} = \tan(\cdot)$ such that $\bar{\gamma}(\alpha) = \frac{x_1}{x_2}$, which demonstrates that $\alpha \leq \frac{x_1}{x_2}$. As a consequence, according to Definition 4, the relation (3) holds. In a similar manner one can show that $\beta \cong \frac{x_2}{x_1}$. But since arctan is a multivalued function, in both cases we can speak only about local equivalence. Even if we consider only the main branch of arctangent, the equivalence still holds only locally. Taking $\hat{\gamma} = x^{-1}$, one can easily show that $\frac{x_1}{x_2} \cong \frac{x_2}{x_1}$. However, the latter equivalence is violated for $x_1 = 0$ or for $x_2 = 0$. As a result, $\alpha \cong \beta$ holds only locally.

One may define the operator $\mathbf{M}$ as follows.

Definition 9. *Given a function $\beta \in \Im_X$, the function $\mathbf{M}(\beta)$ is defined by the following two conditions:*

$$(\mathbf{M}(\beta), \beta) \in \Delta, \quad (\alpha, \beta) \in \Delta \Rightarrow \alpha \leq \mathbf{M}(\beta), \tag{4}$$

where α is an arbitrary function.

The function $\mathbf{M}(\beta)$ exists for every function β and is unique [8], therefore, $\mathbf{M}(\beta)$ may be understood as an operator acting on functions from $\Im_X$. The operator $\mathbf{M}$ can be computed in the following way. Let β be a scalar function such that its composition with the function f can be represented as

$$\beta(f(x,u)) = \sum_{i=1}^{s} a_i(x) b_i(u), \tag{5}$$

where $a_1(x)$, $a_2(x)$, ..., $a_s(x)$ are arbitrary functions and $b_1(u)$, $b_2(u)$, ..., $b_s(u)$ are linearly independent. Then

$$\mathbf{M}(\beta) := a_1 \times a_2 \times \cdots \times a_s. \tag{6}$$

If (5) does not hold, the procedure to compute the operator $\mathbf{M}(\beta)$ is given in the theorem below.

Theorem 1 ([8]). *Let the composition $\beta(f)$ be represented as*

$$\beta(f(x,u)) = \chi(\alpha_1(x), \alpha_2(x), \ldots, \alpha_s(x), u),$$

where the function $\chi \in V \subset \mathbb{R}^{s+1}$ is a function, $\alpha_1(x)$, $\alpha_2(x)$, ..., $\alpha_s(x)$ are the functions satisfying the following condition: there exist inputs $u = c_1$, $u = c_2$, ..., $u = c_r$, such that every function $\alpha_i(x)$ may be expressed via the family of composite functions $\beta(f(x,c_1))$, $\beta(f(x,c_2))$, ..., $\beta(f(x,c_r))$; i.e. the following functional inequality holds:

$$\beta(f(x,c_1)) \times \beta(f(x,c_2)) \times \cdots \times \beta(f(x,c_r)) \leq \alpha_i(x), \\ i = 1, 2, \ldots, s.$$

Then $\mathbf{M}(\beta) \cong \alpha_1 \times \alpha_2 \times \cdots \times \alpha_s$.

One of the properties of the operator $\mathbf{M}$ is

$$\mathbf{M}(\alpha \times \beta) \cong \mathbf{M}(\alpha) \times \mathbf{M}(\beta). \tag{7}$$

3. MAIN RESULT

Theorem 2 ([6,7]). *The system is multi-experiment unobservable if and only if there exists an f-invariant non-injective function α satisfying the condition $\alpha \leq h$.*

Observability criterion. Let the vector function α be the maximal f-invariant function satisfying $\alpha \leq h$. In order to find α, one has to define the sequence of vector functions α_i as follows:

$$\alpha^i = h \times \mathbf{M}(h) \times \cdots \times \mathbf{M}^i(h), \quad i = 1, 2, \ldots. \tag{8}$$

It is easy to note that the functions $\alpha^0 := h, \alpha^1, \alpha^2, \ldots$ form the non-increasing sequence: $\alpha^0 \geq \alpha^1 \geq \alpha^2 \geq \cdots$. The sequence converges, thus for the first k, which satisfied the equivalence relation $\alpha^k \cong \alpha^{k+1}$, we define $\alpha := \alpha^k$. According to Theorem 2, system (1) is observable if the function α is injective, otherwise the system is unobservable.

The following examples show how the observability criterion can be practically used.

[1] This example was suggested by the reviewer C. Moog.

Example 2. Consider the control system

$$\begin{array}{rcl} x_1^+ &=& ux_2, \\ x_2^+ &=& x_3, \\ x_3^+ &=& ux_1+x_4, \\ x_4^+ &=& x_2, \\ y &=& x_3. \end{array} \tag{9}$$

Compute:

$$\begin{array}{rcl} \alpha^0 &:=& h, \\ \alpha^1 &:=& h\times \mathbf{M}(h), \\ \alpha^2 &:=& h\times \mathbf{M}(h)\times \mathbf{M}^2(h), \\ \alpha^3 &:=& h\times \mathbf{M}(h)\times \mathbf{M}^2(h)\times \mathbf{M}^3(h). \end{array}$$

To compute $\mathbf{M}(h)$, one has to find the composition $h(f(x,u)) = ux_1+x_4$. Thus, now in (5) the functions $a_1(x) = x_1$, $a_2(x) = x_4$, $b_1(u) = u$, $b_2(u) = 1$ and according to (6), $\mathbf{M}(h) = x_1\times x_4$. Note that $\mathbf{M}^2(h) = \mathbf{M}(\mathbf{M}(h))$ and due to (7) can be computed as $\mathbf{M}(x_1)\times \mathbf{M}(x_4)$. The composition $x_1\circ f(x,u) = ux_2$ which yields that for $\mathbf{M}(x_1)$ the functions $a_1(x) = x_2$, $b_1(u) = u$ and according to (6), $\mathbf{M}(x_1) = x_2$. The composition $x_4\circ f(x,u) = x_2$ yielding that for $\mathbf{M}(x_4)$ the functions $a_1(x) = x_2$, $b_1(u) = 1$ and according to (6), $\mathbf{M}(x_1) = x_2$. Consequently, $\mathbf{M}^2(h)\cong x_2$. Since $\mathbf{M}^3(h) = \mathbf{M}\left(\mathbf{M}^2(h)\right) = \mathbf{M}(x_2)$, one has to find the composition $x_2\circ f(x,u) = x_3$, which yields that for $\mathbf{M}(x_2)$ the functions $a_1(x) = x_3$, $b_1(u) = 1$ and $\mathbf{M}(x_2) = x_3$. As a result, one obtains:

$$\begin{array}{rcl} \alpha^0 &:=& x_3, \\ \alpha^1 &:=& x_3\times x_1\times x_4, \\ \alpha^2 &:=& x_3\times x_1\times x_4\times x_2, \\ \alpha^3 &:=& x_3\times x_1\times x_4\times x_2\times x_3. \end{array}$$

Note that according to Definition 5, the product of two functions is their maximal bottom and, as a consequence, $x_3\times x_3 = x_3$. (We always keep only the functionally independent components of the result, trying to simplify the result as much as possible.) As a result, $\alpha^3 = x_3\times x_1\times x_4\times x_2$ and $\alpha^2\cong\alpha^3$, yielding that the function $\alpha = \alpha^2 = x_1\times x_2\times x_3\times x_4$ is the maximal f-invariant satisfying $\alpha\le h$. Since α is injective, system (9) is multi-experiment observable.

Example 3. Consider the system

$$\begin{array}{rcl} x_1^+ &=& x_2(u+x_3), \\ x_2^+ &=& u(x_1-x_3), \\ x_3^+ &=& x_2x_3, \\ y &=& x_1-x_3. \end{array} \tag{10}$$

For this system the sequence of functions α_i, $i = 1,2,\ldots$, is the following:

$$\begin{array}{rcl} \alpha^0 &:=& h = x_1-x_3, \\ \alpha^1 &:=& h\times \mathbf{M}(h) = (x_1-x_3)\times x_2, \\ \alpha^2 &:=& h\times \mathbf{M}(h)\times \mathbf{M}^2(h) = (x_1-x_3)\times x_2. \end{array}$$

Obviously $\alpha^1\cong\alpha^2$, consequently $\alpha = \alpha^1 = (x_1-x_3)\times x_2$ is the maximal f-invariant function, satisfying the condition $\alpha\le h$. Since α is not injective, system (10) is multi-experiment unobservable.

We are now ready to present the main result of the paper.

Theorem 3. *If system* (1) *is single-experiment unobservable and the observable space* $\mathscr{O}_\infty$ *is integrable*, *then* (1) *is multi-experiment unobservable.*

Proof. Under the assumption of the theorem there exists the coordinate transformation

$$\varphi = \begin{bmatrix} \tilde\varphi \\ \bar\varphi \end{bmatrix}$$

such that $\tilde z = \tilde\varphi(x)$ are the unobservable coordinates, $\bar z = \bar\varphi(x)$ are the observable coordinates and

$$\begin{array}{rcl} \bar z^+ &=& f^*(\bar z,u), \\ y &=& h^*(\bar z) \end{array} \tag{11}$$

is the observable subsystem [2]. Note that the function $\bar\varphi$ is non-injective. One can write

$$\bar z^+ = \bar\varphi(x^+) = \bar\varphi(f(x,u))$$

and

$$\bar z^+ = f^*(\bar\varphi(x),u),$$

which yields

$$\bar\varphi\circ f(x,u) = f^*(\bar\varphi(x),u). \tag{12}$$

By Definition 7 it follows from (12) that $(\bar\varphi,\bar\varphi)\in\Delta$ and therefore $\bar\varphi$ is an f-invariant function. Additionally, from (1) and (11) one obtains $h(x) = h^*(\bar\varphi(x))$, which by Definition 3 means $\bar\varphi\le h$. Since $\bar\varphi$ is an f-invariant non-injective function and $\bar\varphi\le h$, system (1) is, according to Theorem 2, multi-experiment unobservable. □

Example 4 (Continuation of Example 3)**.** The observable space of this system $\mathscr{O}_\infty = \mathrm{span}_{\mathscr{K}}\{\mathrm{d}x_1-\mathrm{d}x_3,\mathrm{d}x_2\}$ is integrable and the system is single-experiment unobservable. As was shown in Example 3, the system is also multi-experiment unobservable, which confirms the statement of Theorem 3.

The example below demonstrates that the converse statement of Theorem 3 is not valid. Obviously, if the system is multi-experiment unobservable, it is also single-experiment unobservable. However, one cannot say anything about integrability of $\mathscr{O}_\infty$.

Example 5. Consider the system

$$\begin{aligned} x_1^+ &= (1+u)x_3, \\ x_2^+ &= u, \\ x_3^+ &= ux_1 - x_2, \\ x_4^+ &= x_2, \\ y &= x_3. \end{aligned} \tag{13}$$

In order to verify that the system is multi-experiment unobservable, compute the sequence of functions α^i, defined by (8):

$$\begin{aligned} \alpha^0 &:= h = x_3, \\ \alpha^1 &:= h \times \mathbf{M}(h) = x_3 \times x_1 \times x_2, \\ \alpha^2 &:= h \times \mathbf{M}(h) \times \mathbf{M}^2(h) = x_3 \times x_1 \times x_2. \end{aligned}$$

Obviously $\alpha^1 \cong \alpha^2$, consequently $\alpha = \alpha^1 = x_3 \times x_1 \times x_2$ is the maximal f-invariant function satisfying $\alpha \leq h$. Since α is not injective, system (13) is multi-experiment unobservable. The observable space of system (13) is $\mathscr{O}_\infty = \text{span}_{\mathscr{K}} \{u\mathrm{d}x_1 - \mathrm{d}x_2, \mathrm{d}x_3\}$. Although system (13) is also single-experiment unobservable, its observable space $\mathscr{O}_\infty$ is not integrable.

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Ühel ja mitmel eksperimendil põhinevate vaadeldavuse definitsioonide vahelisest seosest diskreetajaga mittelineaarsete süsteemide jaoks

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On uuritud seoseid kahe erineva vaadeldavuse definitsiooni vahel mittelineaarse diskreetajaga juhtimissüsteemi korral. Üks definitsioonidest põhineb ühel eksperimendil ja teine mitmel. Artikli põhitulemus on, et kui süsteem ei ole vaadeldav ühe eksperimendi andmetest ja vaadeldav ruum on täielikult integreeruv, siis ei ole süsteem vaadeldav ka mitme eksperimendi andmetest. Tõestuses on kasutatud uudset funktsioonide algebra aparatuuri. Tegemist on puhtalt diskreetaja nähtustega, sest pidevate analüütiliste süsteemide korral kaks vaadeldavuse mõistet kattuvad ja vaadeldav ruum on alati täielikult integreeruv.