Proceedings of the Estonian Academy of Sciences,
2011, **60**, 1, 38–47
doi: 10.3176/proc.2011.1.04
Available online at www.eap.ee/proceedings

MATHEMATICS

Robust state controller via reflection coefficient assignment

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Received 19 January 2010, accepted 30 March 2010

Abstract. A solution to the robust pole assignment problem via reflection coefficients of polynomials is provided for discrete-time single-input single-output (SISO) and multi-input multi-output (MIMO) linear systems. For SISO systems a robust state controller and the polytopic uncertain plant which is stabilized by this controller have been found. For MIMO systems the problem is solved for an uncertain interval plant.

Key words: discrete-time systems, robust control, stability.

1. INTRODUCTION

In [1] a solution to the robust pole assignment problem via reflection coefficients of polynomials has been provided for discrete-time single-input single-output (SISO) linear systems following the ideas of fixed-order output controller design. In the present paper the reflection coefficient approach is further developed for both SISO and multi-input multi-output (MIMO) robust state controllers. The solution is based on polytopic sufficient stability conditions formulated via reflection vectors of a family of stable polynomials [2].

Several other convex approximations of the stability region such as boxes [3,4], ellipsoids [5,6], polytopes [7,8], or other convex sets [9,10] are widely used in robust control. In [11] a linear Schur invariant transformation with a free parameter was introduced in the discrete polynomial coefficient space, which gives a possibility of generalizing all of these stability conditions by the use of reflection coefficients.

Ideologically, the approach followed in this paper resembles the one given in [12]. In fact, the entire class of controllers attaining polytopic or interval specifications is obtained as a convex set which offers further advantages to the designer. The exact choice of intervals used in specifications is up to the designer.

In order to compare the efficiency of the proposed method with the robust quadratic control, the volumes of stable reflection vector polytopes and stable ellipsoids derived via optimization over linear matrix inequalities (LMIs) [5] are calculated. The volumes of stable reflection vector polytopes are slightly greater than the volumes of stable ellipsoids derived by LMIs.

The reflection coefficients [12] are also known in the literature as Schur–Szegö parameters [13], partial correlation (PARCOR) coefficients [14], or k-parameters [15, ch. 6]. They have been used efficiently in many applications in signal processing [15], system identification [14], and robust control [1].

The main tools for robust state controller design used in this paper are (1) the Luenberger feedback-canonical form [16] of the uncertain plant description and (2) the stable reflection vector polytopes of the closed-loop system [2,11]. For SISO systems we have found a robust state controller which stabilizes the uncertain polytopic plant. For MIMO systems we have found a robust state controller which stabilizes the uncertain interval plant.

The paper is organized as follows. First, in section 2 the solution to the robust reflection coefficient assignment problem has been provided for discrete-time SISO linear systems. In section 3 the same problem has been solved for MIMO systems. A fourth-order example is given for an uncertain interval plant with two inputs and two uncertain parameters.

2. SISO SYSTEMS

Assume that a plant with parametric uncertainties is given. Our goal is to design a state controller so that the closed-loop system is robustly stable and the reflection coefficients of it are assigned in a specific region.

For simplicity, let us consider the problem of state controller design for a SISO plant. Let the plant state space description be given in the feedback canonical form [16]

$$x(t+1) = Ax(t) + bu(t), \tag{1}$$

where

$$A = \begin{bmatrix} o_{n-1} & I_{n-1} \\ & a^T & \end{bmatrix}, \qquad b = \begin{bmatrix} o_{n-1} \\ 1 \end{bmatrix},$$

$x(t)$ is an n-dimensional state vector, $u(t)$ is a scalar input, I_{n-1} is a unit matrix, o_{n-1} is a column vector of zeros with dimensionality $n-1$, and a^T is an n-dimensional row vector of plant parameters.

We are looking for a state controller

$$u = c^T x \tag{2}$$

such that the closed-loop system

$$\begin{gathered} x(t+1) = (A + bc^T)x(t) = Fx(t), \\ F = \begin{bmatrix} o_{n-1} & I_{n-1} \\ & f^T & \end{bmatrix} = \begin{bmatrix} o_{n-1} & I_{n-1} \\ & a^T + c^T & \end{bmatrix} \end{gathered} \tag{3}$$

is stable for a polytopic plant $a \in \mathscr{A}$.

The next theorem defines a state feedback control c in terms of reflection coefficients [13] of the nominal closed-loop system as well as the polytope $\mathscr{A}$ in terms of reflection vectors [2] of the nominal closed-loop system.

Theorem 1. *Assume that the reflection coefficients $k_i(f), i = 1, ..., n$ of the nominal closed-loop system satisfy the conditions*

$$\begin{gathered} k_1(f) \in (-1, 1), \\ k_2(f) = ... = k_{n-1}(f) = 0, \\ k_n(f) \in (-1, 1). \end{gathered}$$

Then the controller

$$c = f - a \tag{4}$$

stabilizes the plant (1) *in the polytope*

$$\mathscr{A} = \mathrm{conv}\{v_i^{\pm}(f) - c\}, \qquad i = 1, ..., n,$$

where $v_i^{\pm}(f)$ are reflection vectors of the characteristic polynomial $f(z)$ of the nominal closed-loop system (3).

To prove this theorem, we have to introduce some basic definitions and relations in the field of reflection coefficients of polynomials.

The reflection coefficients $k_i(f)$, $i=1,...,n$ of a monic polynomial $f(z)$ can be obtained by using backward Levinson's recursion [13]

$$zf^{i-1}(z) = \frac{1}{1-k_i^2(f)}[f^i(z)+k_i(f)f^{i*}(z)], \tag{5}$$

where $k_i(f) = -f_0^i$ and f_0^i denotes the last coefficient of an ith-degree polynomial $f^i(z)$ and $f^{n*}(z)$ is the reciprocal polynomial of $f^n(z)$

$$f^{n*}(z) = f_0 z^n + ... + f_{n-1}z + 1.$$

The stability criterion via a reflection coefficient is as follows [13]: a polynomial $f(z)$ has all its roots inside the unit disk if and only if $|k_i(f)| < 1$, $i = 1,...,n$.

The reflection vectors of a Schur stable monic polynomial $f(z)$ are defined as the end points of the stable line segments $\mathrm{conv}\{f|k_i(f) = \pm 1\}$

$$v_i^{\pm 1} = (f|k_i(f) = \pm 1), i = 1,...,n,$$

where $\mathrm{conv}\{f|k_i(f) = \pm 1\}$ denotes the linear cover obtained by varying the reflection coefficient $k_i(f)$ between -1 and 1, while all the other reflection coefficients are fixed [2].

The linear cover of all the reflection vectors $v_i^{\pm 1}$, $i = 1,...,n$ is called the reflection vector polytope. The following lemma holds [11].

Lemma. *The inner points of a reflection vector polytope of a stable polynomial $f(z)$ with reflection coefficients $k_1(f) \in (-1,1)$, $k_n(f) \in (-1,1)$, $k_2(f) = ... = k_{n-1}(f) = 0$ are stable.*

The proof of Theorem 1 follows immediately from the relations (1)–(3) and the Lemma.

According to Theorem 1, we have two degrees of freedom for choosing the nominal closed-loop system matrix F, i.e. $k_1(f) \in (-1,1)$ and $k_n(f) \in (-1,1)$. There are two reasonable principles for choosing reflection coefficients $k_1(f)$ and $k_n(f)$:

- the volume of the polytope $\mathscr{A}$ must be as great as possible,
- pole placement of the nominal system F must be as good as possible.

First, let $k_1(f) \in (-1,1)$ and $k_2(f) = ... = k_n(f) = 0$. Then $f(z) = z^n - k_1 z^{n-1}$ and its roots are $r_1 = k_1$, $r_2 = ... = r_n = 0$.

Second, let $k_n(f) \in (-1,1)$ and $k_1(f) = ... = k_{n-1}(f) = 0$. Then $f(z) = z^n - k_n$ and the roots of $f(z)$ are placed symmetrically against the origin, whereas $\max|r_i| > |k_n(f)|$.

This means that, according to suggestions for choosing poles of discrete-time systems [17], $k_1(f)$ must be positive and $|k_n(f)|$ must be small in order to obtain a reasonable pole assignment:

$$\begin{aligned} &0 < k_1(f) < 1, \\ &|k_n(f)| << 1, \qquad n > 2, \\ &k_2(f) = ... = k_{n-1}(f) = 0. \end{aligned} \tag{6}$$

The volumes of reflection vector polytopes can be easily calculated by the triangulation method [18]. It is interesting to mention that the volume of reflection vector polytopes does not depend on the first reflection coefficients $k_1(f)$ of the polynomial $f(z)$ for fixed $k_n(f)$ and n with $k_2(f) = ... = k_{n-1}(f) = 0$. The volume of reflection vector polytopes decreases symmetrically by increasing the absolute value of the last reflection coefficients $k_n(f)$ of the polynomial $f(z)$ for n odd and unsymmetrically with a maximum in the positive values of $k_n(f)$ for n even, where $k_2(f) = ... = k_{n-1}(f) = 0$ (Table 1). The maximal volume of reflection vector polytopes decreases by increasing the order n.

Table 1. The volume of stable ellipsoids derived via optimization over LMIs and stable reflection vector polytopes for discrete-time polynomials

	$n=2$	$n=3$	$n=4$	$n=5$
Ellipsoid	2.248	1.479	0.777	0.317
Polytope with $k_1 = \dots = k_{n-1} = 0$, k_n= –0.8	3.6	0.478	0.432	0.035
k_n= –0.4	2.8	1.12	0.784	0.188
k_n= 0.0	2.0	1.333	0.666	0.267
k_n= 0.4	1.2	1.12	0.336	0.188
k_n= 0.8	0.4	0.478	0.048	0.035
Polytope with $k_1 = k_3 = \dots = k_{n-1} = 0$, $k_2 < 0$, $k_n = k_n^{\max}$	4.0	1.773	0.98	0.323

Remark. The volume of stable reflection vector polytopes can be increased by negative values of k_2. The last row in Table 1 gives the volumes for $k_2 \to -1.0$ ($n=2$), $k_2 = -0.3$ ($n=3$), $k_2 = -0.24$ ($n=4$), $k_2 = -0.21$ ($n=5$), and $k_n = k_n^{\max}$, where $k_n^{\max}$ is the k_n which maximizes the volume of stable reflection vector polytopes. In order to compare the efficiency of the proposed method with the robust quadratic control, the volumes of ellipsoids derived via optimization over LMIs are presented in Table 1 (first row) [5]. The volumes of stable reflection vector polytopes are slightly greater than the volume of stable ellipsoids derived by LMIs.

3. MIMO SYSTEMS

Assume that a MIMO plant in the Luenberger feedback-canonical form is given [16]

$$x(t+1) = Ax(t) + Bu(t), \tag{7}$$

where

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & & \vdots \\ A_{m1} & \cdots & A_{mm} \end{bmatrix},$$

$$A_{ii} = \begin{bmatrix} o_{n_i-1} & I_{n_i-1} \\ \multicolumn{2}{c}{a_{ii}^T} \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} o_{n_i-1,n_j} \\ a_{ij}^T \end{bmatrix},$$

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mm} \end{bmatrix},$$

$$b_{ii} = \begin{bmatrix} o_{n_i-1} \\ 1 \end{bmatrix}, \quad b_{ij} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$i,j = 1,\dots,m; \qquad \sum_{i=1}^{m} = n,$$

$x(t)$ is an n-dimensional state vector, $u(t)$ is an m-dimensional input vector, $o_{i,j}$ is an $(i \times j)$-dimensional matrix of zeros.

Theorem 2. *Assume that the reflection coefficients $k_i(f), i = 1,...,n$ of the nominal closed-loop system satisfy the conditions*

$$k_1(f) \in (-1,1),$$
$$k_2(f) = ... = k_{n-1}(f) = 0,$$
$$k_n(f) \in (-1,1).$$

Then the controller

$$C = \begin{bmatrix} C_{11} & \cdots & C_{1m} \\ \vdots & & \vdots \\ C_{m1} & \cdots & C_{mm} \end{bmatrix}, \tag{8}$$

where

$$c_{ij}^T = -a_{ij}^T, \quad j \neq i+1; \quad i,j = 1,...,m,$$
$$c_{i,i+1}^T = [\ 1 \quad 0 \quad \ldots \quad 0\ ] - a_{i,i+1}^T,$$
$$c_m^T = f^T - a_m^T,$$

stabilizes the interval plant (7) $a_{ij} \in (a_{ij}^+, a_{ij}^-)$ *if the coefficient vectors of the closed-loop characteristic polynomials $f_{ij}^{\pm}$ of all the corner plants are placed in the polytope of reflection vectors $v_\alpha^{\pm}(f)$, $\alpha = 1,...,n$ of the nominal closed-loop system f.*

It is easy to see that the closed-loop system matrix

$$F = A + BC$$

is obtained in the companion form

$$F = \begin{bmatrix} o_{n-1} & I_{n-1} \\ \multicolumn{2}{c}{f^T} \end{bmatrix}.$$

Since the last row of F consists of the closed-loop characteristic polynomial coefficients, according to the Lemma, Theorem 2 holds.

Let us now consider a completely controllable uncertain MIMO plant

$$\bar{x}(t+1) = \bar{A}\bar{x}(t) + \bar{B}u(t), \tag{9}$$

where $\bar{a}_{\alpha\beta} \in (\bar{a}_{\alpha\beta}^-, \bar{a}_{\alpha\beta}^+) \in \mathscr{R}$ and $\bar{b}_{\alpha\gamma} \in (\bar{b}_{\alpha\gamma}^-, \bar{b}_{\alpha\gamma}^+) \in \mathscr{R}$, $\alpha, \beta = 1,...,n$, $\gamma = 1,...,m$.

An arbitrary completely controllable MIMO plant can be transformed to the Luenberger feedback-canonical form (7) by a state transformation

$$\begin{gathered} x(t) = T\bar{x}(t), \\ A = T\bar{A}T^{-1}, \\ B = T\bar{B} \end{gathered} \tag{10}$$

if the input matrix $\bar{B}$ has full column rank [16]. In order to design a robust state controller for the interval plant (9), we have to perform the following steps.

1) For the nominal plant $\bar{a}_{\alpha\beta} = (\bar{a}^-_{\alpha\beta} + \bar{a}^+_{\alpha\beta})/2$, $\bar{b}_{\alpha\gamma} = (\bar{b}^-_{\alpha\gamma} + \bar{B}^+_{\alpha\gamma})/2$, $\alpha, \beta = 1, ..., n$; $\gamma = 1, ..., m$ find the transformation matrix T as follows [16].
   - Find the controllability matrix of the nominal plant

$$W = [\ \bar{B} \quad \bar{A}\bar{B} \quad \dots \quad \bar{A}^{n-1}\bar{B}\ ].$$

   - Find the controllability indices of inputs ρ_γ, $\gamma = 1, ..., m$ by inspection of the controllability matrix W. For the full column rank input matrix $\bar{B}$, $\rho_\gamma \geq 1$, $\gamma = 1, ..., m$.
   - According to the controllability indices ρ_γ, transform the controllability matrix W into the form

$$\begin{aligned}\tilde{W} \quad &= [\ \bar{b}_1 \quad \bar{A}\bar{b}_1 \quad \dots \quad \bar{A}^{\rho_1-1}\bar{b}_1 \ \vdots \ \cdots \\ &\cdots \ \vdots \ \bar{b}_m \quad \bar{A}\bar{b}_m \quad \dots \quad \bar{A}^{\rho_m-1}\bar{b}_m\ ].\end{aligned}$$

   - Calculate

$$S = \tilde{W}^{-1}.$$

   - Pick out m rows $s^T_{r_\gamma}$, $\gamma = 1, ..., m$ of the matrix S, where

$$r_\gamma = \sum_{l=1}^{\gamma} \rho_l.$$

   - The transformation matrix T is defined as follows [16]:

$$T = \begin{bmatrix} s^T_{r_1} \\ s^T_{r_1}\bar{A} \\ \vdots \\ s^T_{r_1}\bar{A}^{\rho_1-1} \\ \cdots \\ s^T_{r_m} \\ s^T_{r_m}\bar{A} \\ \vdots \\ s^T_{r_m}\bar{A}^{\rho_m-1} \end{bmatrix}. \tag{11}$$

2) Transform the nominal plant into the Luenberger feedback-canonical form (7) by the transformation T (11).
3) Choose the reflection coefficients k_1 and k_n of the characteristic polynomial $f(z)$ of the nominal closed-loop system according to (6).
4) Taking into account that $k_2 = ... = k_{n-1} = 0$, calculate the vector f of coefficients of the characteristic polynomial $f(z)$ by [2]

$$\begin{bmatrix} f \\ 1 \end{bmatrix} = \begin{bmatrix} f_0 \\ \vdots \\ f_{n-1} \\ 1 \end{bmatrix} = R_n(k_n) \begin{bmatrix} o^T \\ R_{n-1}(k_{n-1}) \end{bmatrix} \cdots \begin{bmatrix} o^T \\ R_1(k_1) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{12}$$

where

$$R_j(k_j) = I_{j+1} - k_j E_{j+1}, \tag{13}$$

I_n is an $n \times n$ unit matrix, E_n is a unit Hankel matrix

$$E_n = \begin{bmatrix} 0 & \dots & 1 \\ . & . & . \\ 1 & \dots & 0 \end{bmatrix},$$

and o^T is a row vector of zeros.

5) Find the robust state controller C according to (7) and (8).
6) Transform the state controller into the initial coordinates by the transformation T (11)

$$\bar{C} = CT^{-1}.$$

7) Calculate the reflection vectors of the characteristic polynomial f according to the definition $v^{\pm}_{\alpha}(f) = (f|k_{\alpha} = \pm 1),\ \alpha = 1, ..., n.$
8) Check the stability of the closed-loop system for the interval plant (9). It is sufficient to check if

$$\{\bar{f}^{\pm}_{\delta}, \in \mathscr{V}, \quad \delta = 1, ..., N\},$$

where N is the number of interval parameters of the plant (9), $\mathscr{V}$ is the polytope of reflection vectors $v^{\pm}_{\alpha}(f), \alpha = 1, ..., n$ and $\bar{f}^{\pm}_{\delta}$ are the coefficient vectors of characteristic polynomials of closed-loop systems for the corner plants $\bar{a}^{\pm}_{\alpha\beta}$, $\bar{b}^{\pm}_{\alpha\gamma}$, $\alpha, \beta = 1, ..., n$, $\gamma = 1, ..., m$

$$\bar{f}^{\pm}_{\delta}(z) = \det(zI_n - \bar{A}^{\pm}_{\alpha\beta} - \bar{B}^{\pm}_{\alpha\gamma}\bar{C}^T).$$

Remark. In principle the entire class of controllers can be obtained as a convex set which stabilizes the nominal plant. The exact choice of reflection coefficients k_1 and k_n from intervals given in specifications is up to designer's making use of the volume of the stable reflection vector polytopes (Table 1) and the pole placement of the nominal system.

Example. Let the interval plant with $n = 4$, $m = 2$, and $N = 2$ be given

$$\bar{A} = \begin{bmatrix} 0 & -1.0 & 0 & 1.0 \pm 0.1 \\ 0.7 \pm 0.05 & -0.4 & 0 & 0.5 \\ 0 & 0 & 0 & -1.0 \\ 0.3 & 0 & -0.2 & 0.8 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 1.0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1.0 \end{bmatrix}.$$

The nominal plant is transformed to the Luenberger feedback-canonical form (7)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.7 & -0.4 & -0.5 & 0.7143 \\ 0 & 0 & 0 & 1 \\ 0.12 & 0.3 & 0.2857 & 0.8 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

by the transformation

$$T = \begin{bmatrix} 0 & 1.4286 & 0.7143 & 0 \\ 1.0 & -0.5714 & 0 & 0 \\ 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}.$$

By choosing $k_1 = 0.5$ and $k_2 = k_3 = k_4 = 0$, we obtain the characteristic polynomial coefficient vector of the nominal closed-loop system from (12)

$$f^T = [\ -0.5 \quad 0 \quad 0 \quad 0\ ]$$

and the controller from (8)

$$C = \begin{bmatrix} 0.7 & 0.4 & 1.5 & -0.7143 \\ -0.62 & -0.3 & -0.2857 & -0.8 \end{bmatrix}$$

or in the initial coordinates

$$\bar{C} = \begin{bmatrix} 0.56 & 0.7 & -1.1 & -0.7143 \\ -0.458 & -0.62 & -0.0414 & -0.8 \end{bmatrix}.$$

The polytope $\mathscr{V}$ of reflection vectors of the nominal closed-loop system is described by the $n \times 2n$ matrix of vertices

$$\begin{aligned} V &= [\ v_1^+(f) \quad v_1^-(f) \quad \dots \quad v_4^+(f) \quad v_4^-(f)\ ] \\ &= \begin{bmatrix} -1.0 & 1.0 & 0 & -1.0 & -0.5 & -0.5 & -0.5 & -0.5 \\ 0 & 0 & -1.0 & 1.0 & 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.0 & 1.0 & 0.5 & -0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.0 & 1.0 \end{bmatrix} \end{aligned}$$

and the corner polynomials of the closed-loop characteristic polynomials for the interval plant by the $n \times 2N$ matrix

$$\begin{aligned} F^{\pm} &= [f(a_{12}^+, a_{21}^+) \quad f(a_{12}^+, a_{21}^-) \quad f(a_{12}^-, a_{21}^+) \quad f(a_{12}^-, a_{21}^-)] \\ &= \begin{bmatrix} -0.16 & -0.16 & -0.16 & -0.16 \\ 0.1305 & 0.1005 & 0.0989 & 0.0689 \\ 0.2188 & 0.1949 & 0.1132 & 0.1017 \\ 0.5808 & 0.5189 & 0.5808 & 0.5198 \end{bmatrix}. \end{aligned}$$

It can be checked that all the corner polynomials $f_\delta^\pm$ are placed in the polytope of reflection vectors $\mathscr{V}$. So the controller $\bar{C}$ stabilizes the interval plant $\bar{A}$, $\bar{B}$. Indeed, the roots of the corner polynomials $f_\delta^\pm$ are as follows:

$$\begin{aligned}\Lambda(f_\delta^\pm) = & \ [\lambda(f(a_{12}^+,a_{21}^+)) \quad \lambda(f(a_{12}^+,a_{21}^-)) \quad \lambda(f(a_{12}^-,a_{21}^+)) \quad \lambda(f(a_{12}^-,a_{21}^-))] \\ = & \begin{bmatrix} 0.6372 \pm 0.7162i & 0.6257 \pm 0.6892i & 0.6409 \pm 0.6752i & 0.6297 \pm 0.6501i \\ -0.5573 \pm 0.5669i & -0.5457 \pm 0.5496i & -0.5609 \pm 0.5963i & -0.5498 \pm 0.5764i \end{bmatrix}.\end{aligned}$$

In order to illustrate the effect of the choice (6) of the reflection coefficient $k_1(f)$ of the nominal closed-loop system, let us choose now $k_1 = -0.8$. By the above procedure we obtain the characteristic polynomial coefficient vector of the nominal closed-loop system

$$\tilde{f}^T = [\ 0.8 \quad 0 \quad 0 \quad 0\ ],$$

the controller

$$\tilde{\bar{C}} = \begin{bmatrix} 0.56 & 0.7 & -1.1 & -0.7143 \\ 0.062 & 0.68 & 0.33 & -0.8 \end{bmatrix},$$

and the corner polynomials of the closed-loop characteristic polynomials for the interval plant

$$\begin{aligned}\tilde{F}^\pm = & \ [\tilde{f}(a_{12}^+,a_{21}^+) \quad \tilde{f}(a_{12}^+,a_{21}^-) \quad \tilde{f}(a_{12}^-,a_{21}^+) \quad \tilde{f}(a_{12}^-,a_{21}^-)] \\ = & \begin{bmatrix} -0.16 & -0.16 & -0.16 & -0.16 \\ -0.3486 & -0.3786 & -0.2762 & -0.3062 \\ -0.4268 & -0.4006 & -0.2959 & -0.2833 \\ -0.7201 & -0.6492 & -0.7201 & -0.6492 \end{bmatrix}.\end{aligned}$$

Unfortunately, the corner polynomials $\tilde{f}_\delta^\pm$ are not placed in the polytope of reflection vectors

$$\begin{aligned}\tilde{V} = & \ [\ v_1^+(\tilde{f}) \quad v_1^-(\tilde{f}) \quad \dots \quad v_4^+(\tilde{f}) \quad v_4^-(\tilde{f})\ ] \\ = & \begin{bmatrix} -1.0 & 1.0 & 0 & 1.6 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0 & 0 & -1.0 & 1.0 & -0.8 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.0 & 1.0 & -0.8 & 0.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.0 & 1.0 \end{bmatrix}.\end{aligned}$$

In fact, the corner polynomials $\tilde{f}_\delta^\pm$ are unstable with roots

$$\begin{aligned}\tilde{\Lambda}(\tilde{f}_\delta^\pm) = & \ [\lambda(\tilde{f}(a_{12}^+,a_{21}^+)) \quad \lambda(\tilde{f}(a_{12}^+,a_{21}^-)) \quad \lambda(\tilde{f}(a_{12}^-,a_{21}^+)) \quad \lambda(\tilde{f}(a_{12}^-,a_{21}^-))] \\ = & \begin{bmatrix} 1.1871 & 1.1741 & 1.1318 & 1.1205 \\ -0.8458 & -0.8348 & -0.8677 & -0.8537 \\ -0.0907 \pm 0.8421i & -0.0897 \pm 0.8089i & -0.0521 \pm 0.8548i & -0.0534 \pm 0.8221i \end{bmatrix}.\end{aligned}$$

4. CONCLUSIONS

A solution to the robust pole assignment problem via reflection coefficients of polynomials is provided for discrete-time SISO and MIMO linear systems. The solution is based on polytopic sufficient stability conditions formulated via reflection vectors of a family of stable polynomials. For SISO systems a robust state controller which stabilizes the uncertain polytopic plant has been found. For MIMO systems the problem is solved for an uncertain interval plant by using the Luenberger feedback-canonical form.

ACKNOWLEDGEMENT

This research was supported through targeted financing of the Estonian Ministry of Education and Research (project SF0140018s08).

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Robustse olekuregulaatori süntees peegelduskoefitsientide kaudu

Ülo Nurges

On esitatud meetod robustse olekuregulaatori sünteesiks peegelduskoefitsientide kaudu nii SISO (ühe sisendi ja ühe väljundiga) kui ka MIMO (mitme sisendi ning mitme väljundiga) diskreetaja lineaarsete süsteemide jaoks. SISO süsteemide puhul on leitud robustne olekuregulaator ja polütoopse määramatusega objekt, mis on stabiliseeritav selle regulaatoriga. MIMO süsteemide puhul on nimetatud probleem lahendatud intervallmudeliga esitatud ebatäpsuse korral.